AN ACHIEVEMENT TEST IN FIRST
YEAR ALGEBRA

by

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Contributions of the Graduate School
Indiana State Teachers College
Number 184

Submitted in Partial Fulfillment
of the Requirements for the
Master of Science Degree
in Education

1934
ACKNOWLEDGMENTS

The writer wishes to express his appreciation for the helpful service rendered by his thesis committee, Dr. W. O. Shriner, chairman, Mr. Orvel E. Strong, Mr. E. L. Abell, and Mr. E. E. Ramsey, in making this study. To Dr. Shriner and Mr. Strong, special gratitude is due for their many helpful suggestions and criticisms.

Sincere thanks are rendered to the Indiana State Mathematics Contest Committee for granting the writer the opportunity of constructing the algebra test which was used in the 1934 Indiana State Mathematics Contest Examinations.

Ivan Horn
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CHAPTER I
I. THE INTRODUCTION

Under the auspices of the Mathematics Section of the Indiana State Teachers Association contests in mathematics are held each year to determine who is the best high-school student in algebra and geometry, respectively. The contest was divided into two parts, the sectional and the final examination. The sectional examination was held over the state in fifteen centers which would best accommodate the mathematics pupils. The final examination was held at Bloomington. Various rules to govern these contest were made by the contest committee.\(^1\)

A. The Problem

It was the purpose of this study to make an achievement test in first year algebra to be used in the State High-School Mathematics Contest, sectional and final, for the school year ending in 1934.

B. Scope of Study

This study was limited to the field of first year algebra. The test was based on the revised edition of Nyberg's *First Course in Algebra*. The limits were set by the Mathematics Contest Committee. The limits for the sectional test, or Form A,

\(^1\)See Appendix A
were from page 1 through page 242. The limits for the final, or Form B, were from page 1 through page 252.

C. General Procedure

The general procedure used in constructing the test and making the study was as follows:

1. The examination of the works of eleven authorities, fourteen current textbooks, and fifteen courses of study in first year algebra was used as a basis for this study.

2. General and specific objectives were selected from these sources to serve as a basis for the test.

3. A careful study was made of the subject matter contained in the revised edition of Nyberg's First Course in Algebra.

4. A study was made of various algebra tests and the technique of making tests.

5. A study was made of the test rules and regulations as made by the Mathematics Contest Committee.²

6. The preliminary test was made in two forms.

7. Each form of the preliminary test was given to high-school and college students taking mathematics.

8. The preliminary test was scored.

9. Final forms of the test were made.

10. Final forms of the test were given.

11. Final forms of the test were scored.

²See Appendix A
12. Data were statistically treated.
13. Conclusions were formulated.

Form A of the preliminary test was given to twelve high-school students taking their third semester of algebra and to twenty-nine college mathematics students, making a total of 41. The time allowed was ninety minutes.

Form B of the preliminary test was given to twelve high-school students who were one and one-half years removed from their third semester of algebra, and fifteen college mathematics students, making a total of 37.

It might be thought that none of these groups was the proper group to take the test. However, when it is recalled that this test was devised for measuring outstanding performance, and that the trial testing was made merely to reveal mathematical inaccu­racies, the use of advanced high-school students and college freshmen is not unwarranted. It was also important that no group eligible for the state contest be used in the preliminary testing.

After Form A had been prepared, it was less difficult to select elements for Form B because of the experience gained from the first test. Fewer students were needed for checking on inaccuracies.

The number was too small for reliable data, but some valuable information was secured. The papers for the preliminary test were used to check the instructions, to see if they were clear and adequate, to help eliminate faulty or poorly stated questions, and to help make the answer keys more nearly complete and satisfactory.
D. Method Used in Collecting Data

After the tests were given at each sectional, the papers were scored by officially appointed committees and returned to Bloomington. Through the co-operation of the Mathematics Contest Committee the papers were secured by the writer for further study.
CHAPTER II

I. OBJECTIVES IN FIRST YEAR ALGEBRA

"In the past, various methods of choosing the subject matter of mathematics have been used. Although much good material has been selected by each of these methods, most if not all of them have their disadvantages. The method of selecting material from various courses of study in use throughout the country, although representative of what is being taught, is open to serious objection, it being well known that such courses generally tend to perpetuate obsolete processes and antiquated business methods and usually fail to be of help in suggesting the thing which ought to be taught.

It is equally true that the best material cannot be secured by making an inventory of the current textbooks in mathematics. They too are frequently guilty of overemphasizing unimportant or obsolete material. It is also true that textbook writers are not always able to suggest newer and better things, and that textbooks are often made merely to fit the demands of certain state syllabi which contain much obsolete and otherwise undesirable material. It is, of course, true that for many schools the textbook is and will continue to be the curriculum, but this does not obviate the necessity for pioneer work on the part of progressive teachers and for freedom to supplement or rearrange the material in a book that is reactionary in treatment, is insufficient in its offering, or admits of improvement in sequence.

The makers of standardized tests in recent years have erred
in including in their work certain exercises and problems that thoughtful teachers everywhere have no desire to see perpetuated in our schools. In fact, many of these undesirable elements were obtained by the makers of the tests from existing courses of study and from textbooks. Thus it is obvious that such tests cannot be used as the sole basis in the selection of desirable material.

We also know that it is not safe to try to determine what mathematics should be taught merely by counting the frequency with which certain mathematical terms are used in a few current editions of newspapers and magazines. Such a method is so unreliable that even those who have pretended to believe in it are abandoning it for more reliable criteria.

Finally, it is fair to say that we cannot satisfactorily formulate the course of study by going out in the world and asking individuals chosen at random what mathematics is useful to them. The fact is that not one of them ever knows just what use he has made of mathematics. Moreover, no one of them has probably given thought to the question of determining how he might have used mathematics profitably if he had only given the matter a little serious attention.

Any one of the above criteria may be of service in selecting the material for a course, but not one or even all will be sufficient for our purpose. If the objectives selected are to meet our modern needs, we must have at least one other criterion, one that is at the same time the result of experience and of good judgment. This last criterion is the opinion of the most expert among the well-trained teachers of mathematics—those who are
able not only to tell how they use the science but also to show how it may be used in the present and in the future for the betterment of mankind." ¹

**Summary.** Courses of study, current textbooks, standardized tests in mathematics, the frequency with which certain mathematical terms are used in newspapers and magazines, and the judgment of individuals chosen at random are not sufficient criteria for selecting the objectives for mathematics. The experience and good judgment of well-trained teachers of mathematics must also be secured to make the list satisfactory and complete.

In order to make this test it was necessary to know what the objectives of algebra are. The methods books in algebra utilize the various sources named above to obtain their list of objectives in mathematics. The textbook is the curriculum in many schools. The courses of study represent what is being taught throughout the country. Therefore, the author has used the opinions of leading educators in the field of mathematics, current textbooks, and courses of study to make the following list of objectives.

A. Opinions of Educators in the Field of First Year Algebra


a. Objectives

(I). Power to think and to do

(A). Powers which are to be developed in all subjects but to which mathematics should make definite contributions

(1). To concentrate on a given task

(2). To carry a task to completion

(3). To use precise and simple language

(4). To systematize and classify given data

(5). To think logically and accurately through a problem

(6). To draw correct inferences

(7). To do neat and accurate work

(8). To do original thinking

(9). To analyze complex situations

(10). To recognize relationships

(11). To discover and plan solutions of problems

(12). To generalize conceptions

(13). To exercise constructive imagination

(B). Powers to be developed which relate
particularly to mathematics in general

(1). To express quantitative facts in mathematical language

(2). To recognize, analyze, and represent mathematical relationships

(3). To discover and formulate mathematical laws

(4). To use graphic representation

(5). To use mathematical methods

(6). To use symbolic notation

(7). To pre-estimate results

(8). To use mathematical concepts, laws, and processes in problems and in quantitative situations

(9). To determine the degree of accuracy possible with measured data

(10). To analyze quantitative situations into simple factors

(11). To do functional thinking

(12). To generalize mathematical concepts and processes

(0). Specific algebraic powers

(1). To understand reading in current literature containing algebraic statements and facts

(2). To use algebraic symbolism

(3). To understand and use the language of algebra
(4). To perform the fundamental operations of algebra with positive and negative numbers
(5). To perform the fundamental operations with monomials and polynomials
(6). To perform the fundamental operations with fractions
(7). To combine and decompose polynomials
(8). To understand and use formulas
(9). To solve formulas for a given letter
(10). To evaluate formulas
(11). To translate verbal statements into formulas
(12). To represent formulas graphically
(13). To express quantitative relationships algebraically
(14). To analyze problem situations
(15). To analyze relationships of quantity
(16). To solve problems by algebraic methods
(17). To solve linear, integral, and fractional equations
(18). To solve simultaneous linear equations algebraically and graphically
(19). To solve quadratic equations algebraically and graphically
(D). Powers to be derived from the graphical method of representing numerical parts and relationships

(1). To understand the meaning of graphical representation

(2). To interpret graphs

(3). To make graphs from given data

(4). To represent equations and formulas graphically

(5). To use the graphical methods of solving equations

(6). To study the properties of functions graphically

(II). Appreciations

(A). Appreciations to be developed by mathematics in general

(1). The contributions mathematics has made to the progress of civilization

(2). The influence of mathematics on the sciences, industries, and everyday life

(3). The relation of mathematics to the pupil's environment

(4). The value of mathematics to other school subjects

(5). The value of mathematics in vocations, business, industry, and architecture
(6). The mathematical modes of thinking
(7). The symbolism of mathematics
(8). The importance of neatness and accuracy
(9). Dependence and relationships of facts in everyday life
(10). The historical development of the mathematical subjects

(B). Appreciations developed in algebra

(1). The value of algebraic notation and symbolism
(2). The value of algebra as a tool for solving problems
(3). The power of the formula and graph

(III). Understandings in algebra

(A). The fundamental concepts of algebra
(B). The laws of algebra with literal numbers
(C). The laws of algebra with positive and negative numbers

(IV). Attitudes

(A). A degree of interest in mathematics which will encourage the pupil to continue in the study
(B). Desire to read mathematical literature growing out of the pleasure to be derived from such reading
(C). Desire to make precise statements
(D). Desire to estimate in advance the solu—
tion of a problem

(E). Desire for thoroughness and clearness
(F). Willingness to concentrate on problems
(G). Desire to analyze complex problem situations

(H). Desire to carry tasks to completion
(I). Desire to do neat written work
(J). Desire to think logically
(K). Attitude of inquiry
(L). Desire to make discoveries
(M). Desire to grow mentally, to improve former records
(N). Desire to constantly improve one's methods, e.g., to prefer the algebraic method of solving problems to the less effective arithmetical method

(O). Desire to concentrate
(P). Desire to understand
(Q). Desire to generalize
(R). Desire to assume responsibility for an assigned task
(S). To consider the study of mathematics a pleasure
(T). Self-confidence in studying mathematics, and in problem-solving

(V). Habits and ideals of:
(A). Carefulness
(B). Persistence
(C). Concentration
(D). Observation
(E). Participation
(F). Neatness in written work
(G). Accuracy
(H). Thoroughness
(I). Clearness
(J). Precision
(K). Interpreting results
(L). Using good language
(M). Checking results

(VI). Skills
(A). Algebraic processes
(B). Making graphs
(C). Solving equations and formulas
(D). Solving problems


a. Aims

(I). Practical aims

(A). An understanding of the language of algebra

(B). A study of the fundamental laws of algebra

(C). The ability to understand and interpret correctly graphic representations of various kinds
(II). Disciplinary aims

(A). The acquisition, in precise form, of those ideas or concepts in terms of which the quantitative thinking of the world is done

(B). The development of ability to think in terms of ideas and concepts

(1). Training in analysis of a complex situation into simpler parts

(2). Recognition of logical relations between interdependent factors

3. Englehardt, Fred, and Haertter, Leonard D., Method of Teaching the First Course in Algebra

a. Abilities

(I). The Formula

(A). Ability to construct a formula

(B). Ability to evaluate a formula

(C). Ability to translate a formula into a rule

(D). Ability to solve practical problems requiring the use of the formula

(II). The simple equation

(A). Ability to enumerate the characteristics of an equation

(B). Ability to solve and check simple equations

(C). Ability to solve a formula for any letter
(D). Ability to express the written statement of an easy problem in equation form and to solve it

(E). An elementary understanding of dependence or functional relationship and variation

(III). Positive and negative numbers

(A). Ability to construct a number-line and to locate various positive and negative numbers on it

(B). Ability to multiply or to divide directed numbers

(C). Ability to add and subtract directed numbers

(D). Ability to spell, use, and give illustrations for the new algebraic words

(IV). Linear and fractional equations

(A). Ability to recognize and describe the properties of a linear or first degree equation

(B). Ability to solve and check the various types of linear equations

(V). Graphs

(A). Ability to read and interpret a statistical graph

(B). Ability to construct a graph of a formula or from a given set of statistical data
(C). Ability to arrange statistical facts
in tabular form

(D). Ability to calculate the average for
relatively easy statistical data

(E). Ability to interpolate data from the
curve of a plotted formula

(VI). Multiplication and factoring

(A). The ability to recognize the principle
involved in raising a quantity to a
power, and multiplying

(B). The ability to multiply a polynomial of
three or more terms by a binomial and
to check results

(C). The ability to write with facility by
inspection the product of such expres-
sions as:

\[(x + y)(x - y), (x + y)^2, (ax + b)
\]
\[(cx + d), (x - y)^2\]

(D). The ability to factor with facility ex-
pressions of the following types:
common monomial factor, trinomial
square, difference of two squares,
quadratic trinomial factorable by trial

(E). Ability to solve certain quadratic equa-
tions by factoring

(VII). Division and fractions

(A). Ability to recognize the principles in-
volved in dividing \(a^m\) by \(a^n\) and in
raising a fraction \( \frac{a}{b} \) to the power \( m \)

(B). The ability to divide a polynomial of four terms by a monomial or binomial

(C). The ability to reduce fractions to simplest form

(D). The ability to perform the fundamental operations as applied to fractions

(E). The ability to find the lowest common denominator in fractions involving polynomials of not more than three terms

(F). The ability to reduce a simple complex fraction to a simple fraction in lowest terms

(VIII). Sets of linear equations

(A). The ability to solve sets of linear equations graphically

(B). The ability to solve sets of linear equations by addition or subtraction

(C). The ability to solve sets of linear equations by substitution

(D). The ability to solve verbal problems which involve sets of linear equations

(E). The ability to check the solution in each type

(IX). Ratio, proportion, and variation

(A). The knowledge that a proportion is a fractional equation which obeys all
the principles which may be applied to equations of this kind

(B). The ability to apply the laws of proportion to the solution of problems involving similar figures, parallel lines and in problems in which a proportional relationship exists

(C). The ability to write an equation of a direct and inverse variation, construct its graph, and discuss intelligently the idea of function

(X). Square root and radicals

(A). The ability to read a table of powers and roots

(B). The ability to find the square root of numbers to the desired degree of accuracy

(C). The ability to perform the fundamental operations with radical expressions

(D). The ability to remove a radical from the denominator of an algebraic expression

(E). The ability to remove a fraction from under a radical

(F). The ability to find the numerical value of fractions containing radicals

(G). The ability to solve simple equations involving radical expressions and to determine their roots
(XI). Indirect measurements
(A). To discover the tangent ratio
(B). To have the pupils formulate their own problems
(C). To use the table of values of trigonometric functions

(XII). The quadratic equation
(A). Ability to solve quadratic equations by completing the square
(B). Ability to use the formula in solving quadratic equations
(C). Ability to solve problems involving quadratic equations
(D). Ability to construct the graph of quadratic equations
(E). Ability to check all answers and roots

4. Lide, Edwin S., Instruction in Mathematics

a. Objectives

(I). Accuracy and facility in the fundamental processes
(A). Computation
(B). Understanding of fundamental laws and operations of algebra
(C). Fundamentals
(D). Tools of problem solving
(E). Practical measurements
(F). Percentage

(II). Knowledge and power to apply mathematical
(I). To teach the recognition of quantities and concepts

(A). Concepts of mathematical law
(B). Number sense
(C). Symbolic notations
(D). Mathematical terms

(III). Specific knowledge useful in life

(A). Graphs
(B). Application of algebra

(IV). Exploration and guidance

(A). Interests and abilities
(B). Prepare for later courses

(V). Disciplinary values

(A). Precision in thought and statement
(B). Self-reliance through check
(C). Logical reasoning
(D). Estimation of results
(E). Quantitative relations
(F). Discrimination between true and false

(VI). Cultural values

(A). Power of applied mathematics
(B). Contribution of mathematics to civilization
(C). Correct habits and attitudes
(D). Interest in nature of community expense

(VII). Specific future needs of well-defined groups

5. Ligda, Paul, The Teaching of Elementary Algebra

ai Objectives

(I). To teach the recognition of quantities and
the nature of relationships

(II). To teach the statement of the relationships in plain English, then the translation of the relating sentences into equations, and the symbolic solution

(III). To teach how to search for implied conditions

(IV). To teach translation into mathematical language

(V). To teach the student to make the relation, to search for significant data, and to formulate the problem

(VI). To develop the ability to discover problems in life work

6. Nunn, T. Percy, *The Teaching of Algebra*

   a. Purpose in teaching mathematics

      (I). To enable the pupil to realize the two-fold significance of mathematical progress

      (II). To teach the importance of mathematics as an instrument of material conquests and of social organization

      (III). To appreciate the value and significance of an ordered system of mathematical ideas

      (IV). To give mental training

7. Schorling, Raleigh, *A Tentative List of Objectives in the Teaching of Junior High School Mathematics*

   a. Objectives

      (I). Attitudes
(A) Basic list

1. Assuming responsibility for correct results
2. Recognizing importance of an "estimated answer"
3. Being interested in and constantly scrutinizing personal growth in the skills of mathematics
4. Appreciating necessity for desirable habits of thrift
5. Being critical of investments
6. Being critical of the extent to which a computed result based on measurement is accurate

(B) Subsidiary list

1. Appreciating the need for the systematization of data in problem solving
2. Appreciating the value of scrutinizing data to seek relations in problem solving
3. Desiring precision of statement
4. Discriminating between true and false
5. Desiring to analyze a complex situation into simpler parts
6. Desiring to discover the general law when examining particulars
(7). Desiring to get at the bottom of a situation
(8). Loving thoroughness and clearness
(9). Appreciating the need for recalling past experience in the solution of a problem

(II). Concepts

(A). Basic list

(1). Ratio
(2). Proportion
(3). Positive and negative numbers
(4). Length
(5). Area
(6). Volume
(7). A surface
(8). An equation
(9). A formula
(10). A graph
(11). An algebraic number
(12). A coefficient
(13). An exponent
(14). A root
(15). A power
(16). A triangle
(17). A square
(18). A rectangle
(19). A parallelogram
(20). A cube
(21). A rectangular prism
(22). A cylinder
(23). An algebraic factor
(24). An algebraic fraction
(25). Pi
(26). Parallel lines
(27). A solution of an equation
(28). Simple interest
(29). Profit
(30). Loss
(31). A root of an equation
(32). Graphic representation
(33). Equality
(34). Similarity
(35). A measurement
(36). Statistical graph
(37). A drawing to scale
(38). Variation
(39). An angle
(40). A circle
(41). A quadratic
(42). A pyramid
(43). An approximation in a measurement
(44). A perimeter
(45). Dependence
(46). A trapezoid
(47). An error in measurement
(48). An average
(49). A cone
(50). A sphere
(51). An indirect measurement
(52). A vertex
(53). A tangent of an angle
(54). Algebra as a tool of science

(B). Subsidiary list

(1). A maximum
(2). A minimum
(3). Direction
(4). Rate
(5). Weight
(6). Velocity

(III). Abilities

(A). Basic list

(1). Graphs

(a). To make and interpret a bar graph
(b). To make and interpret a line graph
(c). To make and interpret a circle graph
(d). To make and interpret a graph of a formula with two unknowns
(e). To use cross section paper to picture amounts of things

(2). Fundamental processes (as applied
to algebra)

(a). To apply fundamental operations to positive and negative numbers

(b). To multiply a monomial by a monomial

(c). To multiply a polynomial by a monomial

(d). To divide a monomial by a monomial

(e). To divide a polynomial by a monomial

(f). To use the law of exponents in multiplication

(g). To use the law of exponents in division

(h). To apply fundamental operations to algebraic fractions of the type of $\frac{a}{b}$

(i). To multiply a polynomial by a polynomial

(j). To divide a polynomial by a polynomial

(k). To find the square root of an arithmetical number by the traditional method

(3). Special products and factors

(a). To give at sight special prod—
ucts of the type: \((ax+b)(cx+d)\)

(b). To give at sight special products of the type: \((ax + cy)^2\)

(c). To give at sight the factors of an algebraic number of the type: \(ax^2 + bx + c\)

(d). To give at sight the factors of an algebraic number of the type: \(a^2 + 2ab + b^2\)

(e). To give at sight the factors of an algebraic number of the type: \(x^2 - y^2\)

(4). Transformation

(a). To transform: \(\sqrt{a^2b} = a\sqrt{b}\)

(b). To transform: \(\sqrt{\frac{a}{b}} = \frac{\sqrt{a}}{\sqrt{b}}\)

(c). To transform: \(\sqrt{\frac{a}{b}} = \frac{\sqrt{a}}{\sqrt{b}}\)

(5). Formulas

(a). To use the formula \(A = bh\) (computing the area of rectangle)

(b). To use the formula \(A = s^2\) (area of square)

(c). To use the formula \(A = \pi r^2\) (area of a circle)

(d). To use the formula \(C = \pi d\) (circumference of a circle)

(e). To use the formula \(V = lwh\) (value of a rectangular prism)

(f). To use the formula \(V = s^3\) (value of a cube)
(g). To use the formula \( V = \pi r^2 h \) 
(value of a cylinder)

(h). To use the formula \( I = prt \) 
(computing interest)

(i). To use the formula \( A = \frac{bh}{2} \) 
(area of triangle)

(j). To use the formula \( V = \frac{bh}{3} \) 
(value of a pyramid)

(k). To use the formula \( S = \pi r^2 + 2\pi rh \) 
(surface of a cylinder)

(l). To solve any of the above formulas for any letter by a proper use of the axioms

(m). To evaluate the above formulas for any letter having given the values of all other letters that occur in those formulas

(n). To translate a practical rule of procedure in arithmetic by a formula

(o). To solve the formula \( a^2 + b^2 = c^2 \) for either \( a \) or \( b \)

(p). To use the formula \( V = \frac{4}{3}\pi r^3 \) 
(volume of a sphere)

(q). To use the formula \( A = \frac{h(B + b)}{2} \) 
(area of a trapezoid)

(r). To translate a formula (difficulty not to exceed \( F = \left( \frac{9}{5}C + 32 \right) \)
(a). To express by an equation between two numbers that is stated in the form of an English sentence (difficulty not to exceed "To find the Fahrenheit reading corresponding to some known Centigrade reading multiply it by $\frac{9}{5}$ and add 32")

(b). Equation

(a). To use the addition, subtraction, multiplication and division axioms in the solution of an algebraic equation

(b). To clear an equation of fractions

(c). To interpret a negative number as an answer when the unknown forms one member of the equation

(d). To find the fourth number in a proportion when three numbers are given

(e). To solve simultaneous equations in two unknowns by the addition and subtraction method

(f). To solve an equation in one unknown containing fractions with binomial denominator
(g). To solve simultaneous linear equations in two unknowns by the graph method

(B). Subsidiary list

(1). Graphs
   (a). To express direct variation by a graph
   (b). To solve a quadratic equation by the graphic method

(3). Recognizing non-essential data in problem solving

(3). Formulas
   (a). To use the formula \( V = \frac{\pi r^2 h}{3} \)
      (volume of a cone)
   (b). To use the formula \( S = 4\pi r^2 \)
      (surface of a sphere)
   (c). To write a formula showing how two related numbers change

(IV). Information

(A). Basic list

(1). Fundamental processes
(2). Per cents
   (a). Per cent means hundredths
   (b). How to compute profit on cost
   (c). Interest = principal \( \times \) rate \( \times \) time
   (d). Meaning of commission

(3). Algebra
(a). How to change form of algebraic fraction without changing the value
(b). Meaning of unknown
(c). Meaning of binomial
(d). Meaning of opposite quantities
(e). Meaning of members of an equation
(f). Meaning of base
(g). Meaning of radical
(h). Meaning of trinomial
(i). Meaning of constant
(j). Meaning of variable
(k). Meaning of degree of a number
(l). Meaning of degree of a term
(m). Meaning of like terms
(n). Meaning of linear function
(o). Meaning of descending powers
(p). Meaning of highest common factors
(q). Meaning of reciprocal
(r). Meaning of satisfy
(s). Meaning of similar terms
(t). Meaning of identity
(u). Meaning of partial products
(v). Meaning of polynomial
(w). Meaning of origin

(B). Subsidiary list

(1). Fundamental processes
(a). Meaning of "correct to n" significant figures
(b). Order of signs in processes
(2). Algebra
(a). Meaning of rational
(b). Meaning of mixed expression

a. Aims and values

(I). Practical value
(A). The use of mathematics in science
(B). The influence of mathematics on life
(C). The value of mathematics to the individual

(II). Disciplinary value
(A). Exercise of the reasoning power because of the simplicity, accuracy, certainty of results, originality, similarity to the reasoning of life, and amount of reasoning

(III). Minor advantages
(A). Development of power of concentration
(B). Development of the constructive imagination
(C). Growth of mental self-reliance
(D). Development of character
(E). Increased ability to use English correctly
(F). Increase in general culture


a. Basis for choosing objectives

(I). The intrinsic worth of mathematics itself

(II). The social needs of the people, especially those in the local community itself

(III). The interests of the pupils

b. Great central mathematical objectives

(I). Introduction to the general range of elementary mathematics

(II). Some appreciation of the power of mathematics

(III). The increase of certain powers

(IV). Fostering the study of mathematics

c. General objectives

(I). Establishing habits of:

(A). Neatness and method

(B). Thinking

(C). Moral conduct

(D). Character

(II). Exercise in fundamental modes of thought

(A). Simplicity of language

(B). Accuracy in reasoning

(C). Originality in thought

(III). Appreciation of mathematics as a useful art

(A). Power of expressing data systematically

(B). Scrutinizing these data in solving problems
(C). Organizing these data as an aid to memory

(D). Succinctness of mathematical statements of laws and formulas

(E). The equation as an aid in using formulas

(IV). Appreciation of mathematics as a science

(A). Significance of symbolism

(B). Interlacing of branches

(C). The relation of mathematics to allied subjects

(D). The relation of formulas to general truths

(E). The eternal verities of mathematics

(F). The universality of functional relationships

(G). The value of mathematics for its own sake

(H). Relation to the Infinite

(I). Recreational side of mathematics

(J). Connection with art

(K). Rhythm of mathematics

(L). Relation to nature

(V). Appreciation of the historical growth of mathematics

(A). Mathematics a moving stream

(B). Significance of our numerals

(C). Growth of the fraction

(D). Displacement of compound numbers

(E). Merging of decimals and per cents

(F). Systems of measure
(G). Change from rule to formula
(H). Development of symbols
(I). Growth of applied mathematics

(VI). Attitudes of mind to be developed

(A). Responsibility for accuracy
(B). Satisfaction with thorough work and precision of statement
(C). Common-sense estimates of results
(D). Dissatisfaction with vague results
(E). Recognition of irrelevant data
(F). Discrimination between the true and the false
(G). Desire to analyze a complex situation into its components
(H). Self-reliance in attacking a problem
(I). Desire to search out the truth
(J). Constant seeking for applications of mathematics in daily life
(K). Interest in developing skill in mathematics
(L). Desire to generalize

(VII). Ideals to be cultivated

(A). Devotion to truth
(B). Originality in action
(C). Neatness in solutions
(D). An appreciation of our relation to the universe
(E). Respect for one another as small por-
tions of the infinite

(F). Regard for the beautiful

(G). Loyalty to the family, the community, and the State

(H). Respect for a good reputation

d. Abilities in algebra

(I). Formulas

(A). Ability to discover rules and to translate these into formulas

(B). Ability to translate formulas into rules

(C). Ability to evaluate formulas

(D). Ability to derive one or more formulas from a given formula

(E). Ability to represent by a graph any simple formula

(F). Ability to understand the dependence of one quality upon another

(G). Ability to work with ordinary simple formulas

(II). The graph of a mathematical law

(A). Ability to understand and use directed numbers

(B). Ability to represent points by means of the usual coordinate system

(C). Ability to understand the types \( y = kx \) and \( xy = k \)

(D). Ability to compare graphs drawn with respect to the same axes
(E). Ability to interpolate and extrapolate
(F). Ability to interpret the graph of a simple formula
(G). Ability to interpret intersections with the axis
(H). Ability to understand the relations between the variables in certain special cases
(I). Ability to read a compound-interest graph
(J). Ability to use the graph of \( y = x^2 \)
(K). Ability to use the graph of an equation like \( y = 3x + 1 \)
(L). Ability to use and to interpret the graph of an equation like \( y = 2x^2 + x - 10 \)
(M). Ability to discover a maximum or a minimum in a graph
(N). Ability to read values from a graph
(O). Ability to use graphs in allied fields of science
(P). Ability to distinguish between the significance of a graph of statistics and that of a mathematical law
(Q). Ability to express a ratio graphically

(III). Linear equations in one unknown
(A). Ability to translate a verbal problem into an equation
(B). Ability to translate an equation into words
(C). Ability to solve equations of the type
\[ ax + b = c \]

(D). Ability to understand the significance
of the graph of an equation of the
type \[ y = ax + b \]

(E). Ability to solve linear equations contain­
taining common or decimal fractions

(F). Ability to use equations in solving
problems

(G). Ability to interpret artificial numbers
in a result

(IV). Directed numbers

(A). Ability to represent directed numbers
graphically

(B). Ability to use directed numbers practi­
cally

(C). Ability to add or to subtract directed
directed

(D). Ability to multiply by a positive or by
a negative number

(E). Ability to divide by a positive or by a
negative number

(F). Ability to use directed numbers in graphs

(G). Ability to use directed numbers in form­
ulas

(H). Ability to remove either one or two sets
of parentheses

(I). Understanding of the double use of the
signs + and –
(V). Operations with polynomials

(A). Addition, subtraction, multiplication, and division

(B). Multiplication of a binomial by a monomial

(C). Multiplication of any polynomial by a monomial

(D). Multiplication of a binomial by a binomial

(E). Division of a binomial by a monomial

(F). Division of a polynomial by a monomial

(G). Division of a polynomial by a binomial

(H). Using symbols of aggregation

(I). Understanding the value of factoring

(J). Understanding factoring as the inverse of multiplication

(K). Ability to remove a monomial factor

(L). Ability to factor the general quadratic trinomial

(M). Ability to factor expressions like $a^2 - b^2$

(N). Understanding why division by 0 is not permitted

(O). Understanding the significance of complete factoring

(P). Ability to apply the laws of exponents

(Q). Understanding of zero, negative, and fractional exponents

(R). Ability to check all results
(S). Understanding of the generality of algebra

(VI). Fractions
(A). Understanding that a fraction means division
(B). Understanding the principle of signs
(C). Ability to reduce a fraction to lowest terms
(D). Ability to perform other reductions
(E). Ability to simplify such complex fractions as may be needed in work with practical formulas
(F). Understanding of the connection between arithmetical and algebraic fractions
(G). Ability to check all results

(VII). Fractional equations
(A). Ability to clear an equation of fractions
(B). Ability to solve numerical equations containing fractional coefficients
(C). Ability to solve fractional equations
(D). Ability to derive one formula from another
(E). Understanding the generality of a literal equation
(F). Ability to solve for a constant an equation like \( y = ax + b \)
(G). Ability to evaluate formulas involving fractions
(H). Ability to check all results

(VIII). Ratio, proportion, and variation
(A). Understanding a ratio as an abstract quotient
(B). Understanding a proportion as an equality of ratios
(C). Understanding that a ratio, written in fractional form, is subject to all the laws of fractions
(D). Understanding variation as related to ratio
(E). Ability to solve problems in variation
(F). Understanding more fully the idea of function

(IX). Simultaneous linear equations
(A). Understanding types
(B). Ability to choose the best method
(C). Ability to solve applied problems
(D). Ability to check all results

(X). Powers and roots
(A). Understanding necessary terms
(B). Seeing the practical need for roots
(C). Ability to find the square root of a number
(D). Ability to find the square root of a polynomial
(E). Understanding how far to carry a square root
(F). Understanding common exponential symbolism

(G). Application of square root

(H). Solving of radical equations

(XI). Quadratic equations

(A). Ability to construct and interpret the graph of a quadratic function

(B). Understanding of the terms "complete" and "incomplete"

(C). Ability to solve a quadratic equation by factoring

(D). Ability to solve by completing the square

(E). Ability to find the maximum or minimum of value of a quadratic function

(F). Ability to solve applied problems

(G). Ability to check the solutions

(XII). Numerical trigonometry

(A). General purpose

(1). To show the pupils the significance of indirect measure

(2). To show the significance of trigonometry in the life of today

(B). Necessary functions

(1). Four important functions
   (a). Sine
   (b). Cosine
   (c). Tangent
   (d). Cotangent

(2). Definition of functions
(3). Introduction to the functions
(4). Ability to find the functions
(5). Ability to use any convenient tables

(C). Applications

(1). Ability to apply the subject to solving right triangles

(2). Instruments needed

(3). Ability to check

10. Stone, John C., and Mallory, Virgil S., First Year Algebra Manual For Teachers

a. Objectives

(I). General

(A). To think in an orderly and logical manner

(B). To base reasoning on sufficient evidence

(C). To analyze situations for causal relations

(D). To strip pertinent elements from a complex situation

(E). To learn to picture relations in a graphic manner

(F). To understand functional relationships

(G). To know the effect a single varying element may have upon another element

(H). To express thoughts in quantitative terms

(I). To develop attitude of:

(1). Independence
(2). Resourcefulness
(3). Inquiry after truth
(4). Insistence on accuracy
(5). Quantitative thinking
(6). Neatness of work

(J). To apply in more advanced fields the power acquired

(K). To encourage pupils to transfer arithmetical skill and abilities to algebra

(L). To use abilities and skills they have developed

(M). To develop the ability to solve problems

(II). Specific

(A). The formula

(1). Meaning and use of the formula
(2). Making and evaluating formulas
(3). Adding and subtracting literal numbers
(4). Powers and exponents
(5). Use and need of parentheses
(6). Representing a formula graphically
(7). Using formulas to give equations
(8). Solving equations
(9). Changing the subject of a formula
(10). Showing dependence by formulas

(B). Directed numbers

(1). Meaning and use of signed numbers
(2). Locating points on a graph
(3). Adding, subtracting, multiplying, and dividing signed numbers

(4). Adding and subtracting of polynomials

(5). Removing parentheses in equations

(6). Translating a problem into an equation

(C). Multiplication and division

(1). Multiplying monomials

(2). Multiplying a polynomial by a monomial

(3). Multiplying polynomials

(4). Dividing monomials

(5). Dividing a polynomial by a monomial

(6). Dividing one polynomial by another

(D). Special products and factors

(E). Fractions

(1). Reducing fractions to their lowest terms

(2). The signs of a fraction

(3). Multiplying fractions

(4). Dividing fractions

(5). Changing fractions to mixed expressions

(6). Changing a fraction to a given denominator

(7). Changing two or more fractions to a common denominator

(8). Adding and subtracting fractions
(9). Solving equations
(10). Solving literal equations

(F). Linear equations
(1). Graphical representation of a linear equation in two unknowns and graphical solution of a system
(2). Inconsistent and indeterminate systems
(3). Elimination by addition or subtraction
(4). Elimination by substitution
(5). Use of two unknowns to solve a problem
(6). Problems involving the digits of a number
(7). Problems involving opposite rates
(8). Simultaneous equations whose unknowns are used as the denominators of fractions

(G). Powers, roots, and radicals
(1). Power of monomials
(2). The meaning of roots and radicals
(3). Fractional exponents
(4). Method of finding the square root of a number
(5). Applications of square root
(6). Irrational numbers
(7). Placing the coefficient of a radical under the radical sign
(8). The square root of a fraction and rationalizing the denominator
(9). Addition and subtraction of radicals
(10). Multiplication and division of radicals
(11). The square root of trinomials
(12). Changing a quadratic equation to the form $x^2 + ax = b$
(13). Solving quadratic equations by completing the square
(14). Solving a quadratic when the root is irrational
(15). The general quadratic equation
(16). Making and using a formula for solving quadratics
(17). Special types of quadratics
(18). Graphs and quadratic equations

(H). Ratio, variation, proportion
(1). The meaning of ratio
(2). The meaning of constants, variables, and functions
(3). The language of variation
(4). Inverse variation
(5). The language of proportion
(6). Finding heights and distances by proportion
(7). Problems of the lever
(8). Trigonometric ratios
(9). The numerical value of trigonometric ratios
(10). Use of trigonometric ratios to find heights and distances
(11). Areas of triangles found by trigonometric ratios
(12). Use of trigonometric ratios giving one-sixth of a degree
(13). Finding the value of ratios not given in the table


a. General objectives

(I). Acquaint the pupils with a knowledge of the language of algebra

(II). Pay attention to the formula and to the statistical graph

(III). Give preparatory work in various subjects that contain algebra to those students whose capacities will probably lead them to fields of physics, electricity, etc.

(IV). Develop mastery of proportion and other easy forms of the equation

(V). Extend the field of application of the construction of formulas as well as their evaluation

(VI). Develop the art of criticism as applied to graphs

(VII). Encourage the presentation of laws by means
of mathematical graphs

(VIII). Use the function concept when advantageous, but with economy

(IX). Form bonds which provide the pupil with useful algebraic abilities in any case, and stimulate him to understand principles so far as he has the ability

B. Objectives in First Year Algebra

From Current Textbooks

1. Barber, Harry C., Everyday Algebra
   a. General objectives
      (I). To encourage straight thinking
      (II). To strengthen computational abilities
      (III). To develop a checking habit
   b. Specific objectives
      (I). What algebra does for us and how
         (A). To show the utility of elementary algebra
         (B). To teach the solution of simple linear equations
         (C). To lay the foundation for a sound method of problem solution
         (D). To define algebra as a useful, interesting, rational subject
      (II). Equations containing fractions
         (A). To teach the simple uses of proportion,
and other easy fractional equations, together with the necessary multiplication of monomial factors; and meanwhile to keep algebra in the realm of things understood rather than in the realm of mechanical rule

(III). Graphs, pairs of linear equations, negative numbers

(A). To illustrate the functional relation as expressed by words, tables, graphs, and equations

(B). To enlarge in the mind of the pupil the meaning of the minus sign, and to show how the addition, subtraction, multiplication, and division of signed numbers can be understood and explained

(C). To teach the solutions and uses

(IV). Quadratic equations, multiplication, division, factoring

(A). To teach the solution of quadratic equations by factoring and by a formula

(B). To enlarge the pupil's control over the four processes

(C). To strengthen the pupil's growing ability to write the shorthand of algebra and to translate it into words

(D). To show that algebra has broader powers than arithmetic

(E). To show something of the nature of mathematical proof
(V). The useful and fundamental parts of algebra
(A). To provide the pupil with much interesting material with which to test his skill and understanding
(B). To review the more useful parts of elementary algebra

(VI). Trigonometry
(A). To broaden the pupil's conception of the utility of mathematics
(B). To strengthen the pupil's control of practical numerical computation
(C). To teach the use of tables
(D). To show the method of studying one triangle and thereby obtaining useful information about other similar triangles
(E). To show how algebra, geometry, and trigonometry work together in the solution of practical problems

(VII). Geometry
(A). To teach some of the facts about reasoning which a ninth-year pupil should know
(B). To enlarge his acquaintance with geometric facts and their uses
(C). To teach convenient methods of using radicals in numerical computations

(VIII). Commercial mathematics: statistics
(A). To review commercial arithmetic and
graphs
(B). To teach simple and useful ideas about handling of statistics
(C). To encourage pupils in commercial courses to lay a good mathematical foundation for their future work

(IX). More about the mechanics of algebra
(A). To provide advanced work for pupils who already understand much about the nature and purpose of elementary algebra
(B). To develop skill in the kinds of manipulation frequently demanded by formal examinations

(X). Practical computation
(A). To interpret for the ninth school year the very important reform in computation which has occurred during the past two decades
(B). To increase the pupil's ability to perform numerical computations in a common-sense manner

2. Betz, William, *Algebra For Today*

a. General objectives

(I). To make elementary algebra more meaningful
(II). To give greater emphasis to the really significant and basic ideas of the subject
(III). To make problem-solving more than a mere incident
(IV). To make the technique simpler and more purpose-
ful
(V). To blend the old and the new
(VI). To stress the new ideals in secondary mathematics
(VII). To provide an adequate background for both interest and effort

b. Specific objectives

(I). How letters are used in algebra
(A). The use of letters in algebra
(B). The meaning of certain new words
(C). The rule of similar terms
(D). The rule of order for addition
(E). The rule of grouping for addition
(F). How to write certain formulas
(G). How to evaluate formulas and algebraic expressions
(H). How to combine letters and numbers by the four fundamental operations

(II). The use of the formula
(A). To learn important formulas used in mensuration
(B). To learn important formulas used in business
(C). To evaluate these formulas
(D). To express the meaning of these formulas in words
(E). To apply these formulas
(F). To obtain formulas from verbal statements
(G). To obtain formulas from tables
(III). The equation

(A). To solve equations by inspection
(B). To solve equations by rule
(C). To check the solution of an equation
(D). To learn the use of the fundamental axioms
(E). To learn the meaning of important terms, such as equality, identity, equation, root, solution, members of an equation

(IV). Problems

(A). To solve simple number problems
(B). To solve simple geometric problems
(C). To solve business problems
(D). To solve other applied problems

(V). Graphs

(A). To make and interpret various kinds of statistical graphs
(B). To know the important uses of graphs
(C). To make and use the graph of a formula
(D). To solve problems graphically

(VI). Signed numbers

(A). The meaning and important uses of positive and negative numbers
(B). To picture signed numbers
(C). To apply the four fundamental operations to signed numbers

(VII). Fundamental operations
(A). To know the meaning of base, exponent, and power

(B). To combine algebraic expressions by addition and subtraction

(C). To understand the laws of order, grouping, exponents, and distribution for multiplication

(D). To multiply a monomial by a monomial, a polynomial by a monomial, and a polynomial by a polynomial

(E). To understand the laws of exponents, and distribution for division

(F). To divide a monomial by a monomial, a polynomial by a monomial, and a polynomial by a polynomial

(VIII). Equations of the first degree in one unknown

(A). To solve equations of the first degree

(B). To transform an equation by addition or subtraction

(C). To transform an equation by changing signs

(D). To solve percentage problems, motion problems, mixture problems, and other applied problems

(IX). Equations having two unknowns

(A). To draw the graph of a linear equation

(B). To solve pairs of linear equations by various methods

(C). To use two letters in solving number-
relation problems, digit problems, and other applied problems
(X). Special products and factoring
(A). To find the following special products
(1). The product of any two binomials
(2). The square of any binomial
(3). The product of the sum and the difference of two terms
(B). To deal with the following cases in factoring
(1). Removing a common monomial factor
(2). Factoring a trinomial of the form $ax^2 + bx + c$
(3). Factoring a trinomial square
(4). Factoring the difference of two squares
(C). To use special products and factoring in solving equations and problems
(XI). Fractions
(A). To change fractions to equivalent fractions by applying the Fundamental Principle of Fractions
(B). To find the l. c. d. of two or more fractions
(C). To combine fractions by addition, subtraction, multiplication, or division
(D). To change the signs of a fraction
(E). To simplify complex fractions
(XII). Fractional equations
(A). To solve problems in which fractional equations are necessary
(B). To solve a literal fractional equation for any letter appearing in the equation in terms of the other letters
(C). To transform formulas containing fractions

(XIII). How quantities change together
(A). The importance of related changes in everyday life
(B). The expression of numerical relationships and cases of dependence
(C). The meaning of variable, constant, independent variable, dependent variable, function
(D). The use of ratios in numerical comparisons
(E). The use of proportions in the study of similarity and in other applied problems
(F). The meaning and use of various types of variation

(XIV). Numerical trigonometry
(A). The meaning and importance of indirect measurement
(B). The direct measurement of lines and angles
(C). Important facts about triangles
(D). The solution of trigonometric problems
by scale drawing

(E). The meaning and use of the tangent, the sine, and the cosine

(F). The solution of trigonometric problems by computation

(XV). Square root and radicals

(A). Finding the square root of a number
   (1). From a table
   (2). From a graph
   (3). By computation

(B). The meaning of such terms as radical, index, radicand, root, surd

(C). Finding the product and the quotient of radicals of the same degree

(D). Rationalizing the denominators of radicals involving fractions

(E). Finding the sum or the difference of similar radicals

(F). Solving a radical equation

(G). Transforming and evaluating formulas containing radicals

(XVI). Equation of the second degree in one unknown

(A). Solving an incomplete quadratic equation by:
   (1). Factoring
   (2). The square-root method

(B). How to solve a complete quadratic equation by:
   (1). Graph
(3). Factoring
(3). Completing the square
(4). The quadratic formula
(5). Solving problems involving quadratic equations

3. Engelhardt, Fred, and Haertter, Leonard D., *First Course in Algebra*

a. Specific objectives
   (I). To get acquainted with the nature and uses of mathematics
   (II). To make and use formulas
   (III). The simple equation
      (A). To teach the types of equations
      (B). To study independent and dependent variables
   (IV). To apply the four fundamental operations to positive and negative numbers
   (V). Linear and fractional equations
      (A). To solve linear equations
      (B). To solve equations containing parentheses
      (C). To solve problems containing fractional equations
   (VI). Problems
      (A). To teach the important points to remember in the solution of problems
      (B). To solve work, geometry, and mixture problems
   (VII). Make and read various kinds of graphs
(VIII). Multiplication and factoring
   (A). To teach the laws of exponents
   (B). To multiply polynomials by monomials
   (C). To teach method of proceeding in finding a monomial factor
   (D). To multiply polynomials by binomials
   (E). To find the products of binomials
   (F). To teach factoring
   (G). To teach how to find special products
   (H). To solve quadratic equations by factoring

(IX). Division and fractions
   (A). To teach the laws of exponents in division
   (B). To divide by monomials
   (C). To divide a polynomial by a polynomial
   (D). To use algebraic long division
   (E). To reduce fractions
   (F). To apply the four fundamental operations to fractions
   (G). To find the lowest common multiple

(X). Fractional and literal equations
   (A). To know the important facts to remember in solving fractional equations
   (B). To solve fractional and literal equations

(XI). To use various methods in solving sets of linear equations
(XII). Ratio, proportion, and variation

(A). To know the meaning of ratio, proportion, and variation
(B). To use ratio, proportion, and variation
(C). To solve problems containing ratio, proportion, and variation

(XIII). Square root and radicals

(A). To use fractional exponents to express roots
(B). To use tables in finding roots
(C). To find a root of a monomial expression
(D). To find the square root of numbers
(E). To solve problems involving square roots

(XIV). Indirect measurement

(A). To use three methods in doing indirect measurement
(B). To find the tangent
(C). To use a table of sines, cosines, and tangents

(XV). Quadratic equations

(A). To solve quadratic equations by completing the square and by a formula
(B). To graph quadratic equations
(C). To solve sets of equations, one linear and one quadratic

4. Hart, W. W., First Book in Algebra

a. Specific objectives

(I). Foundations for algebra
(A). Formulas

(B). To get acquainted with the mathematical vocabulary

(C). Graph formulas

(II). Equations

(A). To learn the various methods of solving equations

(III). Positive and negative numbers

(A). To familiarize the pupils with positive and negative numbers and their uses in daily life

(IV). Addition and subtraction

(A). To add positive and negative numbers and algebraic expressions

(B). To subtract positive and negative numbers

(C). To use and remove parentheses in addition or subtraction

(V). Multiplication and division

(A). To multiply signed numbers

(B). To divide signed numbers

(VI). Simple equations

(A). To understand what a graph is and how to make and interpret various kinds


a. General objectives

(I). To give the proper training for those going to college
(II). To give the proper training for those not going beyond the first year of algebra

(III). To make algebra interesting to the student by:

(A). Closely correlating the new subject with the old

(B). Making it seem real

(C). Giving well-graded and practical exercises that the student can learn without discouragement

(IV). To master subjects

(V). To form correct habits of study

b. Specific objectives

(I). Introductory review

(A). To eliminate student difficulties with fractions, decimals and percentages by starting with a review of those fundamentals

(II). Statistical graphs

(A). To familiarize the students with the use and making of statistical graphs

(III). Literal numbers and equations

(A). To teach some of the terms of algebra

(B). To teach equations

(1). To subtract, add, multiply and divide each member of an equation

(2). To solve equations

(3). To teach transposition

(4). To use the equation to solve problems
(C). To make formulas from written statements
(D). To solve problems using formulas

(IV). Negative numbers
(A). To teach the meaning and use of positive and negative numbers
(B). To use the four fundamental operations involving signed numbers

(V). Adding and subtracting polynomials
(A). To add and subtract polynomials
(B). To solve equations containing symbols of grouping
(C). Practice in the use of algebraic language

(VI). Multiplying polynomials
(A). Multiplying a polynomial by a monomial
(B). Multiplying one polynomial by another
(C). Products of two binomials by inspection
(D). Powers of monomials
(E). Equations involving multiplication of polynomials

(VII). Dividing polynomials
(A). Dividing a polynomial by a monomial
(B). Dividing one polynomial by another

(VIII). Fractions and equations containing fractions
(A). Addition, subtraction, multiplication, and division of fractions
(B). To solve equations containing fractions
(C). To solve literal equations
(D). To evaluate and solve formulas

(IX). Ratio, proportion, similar polygons, and
numerical trigonometry

(A). To teach ratio and proportion

(B). To teach the properties of similar polygons

(C). To study sines, cosines, and tangents of angles

(X). Powers and roots

(A). To teach the meaning of powers and roots

(B). To find powers and roots of numbers

(XI). Radicals and pure quadratic equations

(A). To teach the meaning and use of radicals

(B). To teach quadratic surds

(C). To simplify quadratic radicals

(D). To add and subtract surds

(E). To solve pure quadratic equations

(F). To solve problems in geometry

(G). To make quadratic graphs

(XII). To complete quadratic equations

(A). Solve quadratic equations by:
   
   (1). Completing the square
   
   (2). Using the formula

(B). Solve problems involving quadratic equations

(XIII). Systems of equations in two unknowns

(A). To teach the various kinds of equations

   (1). Indeterminate

   (2). Dependent

   (3). Independent
(4). Simultaneous
(5). Inconsistent equations

(B). Solve simultaneous first degree equations either by the addition and subtraction, the substitution, or the graphic method

(XIV). Factoring

(A). To teach the uses of factoring
(B). To multiply a polynomial by a monomial
(C). Find the prime factors of polynomials
(D). Factor the difference of two squares
(E). Factor a quadratic trinomial

(XV). Fractions and equations

(A). Reduce fractions to lowest terms
(B). To apply the four fundamental processes to fractions
(C). To solve equations by factoring
(D). To solve literal equations


a. Specific objectives

(I). To acquaint the pupil with the meaning and usage of literal expressions
(II). To apply the four fundamental operations to positive and negative numbers
(III). To make and interpret different kinds of graphs
(IV). Addition and subtraction of polynomials

(A). To teach the addition and subtraction of
polynomials
(B). To use the parentheses in addition and subtraction of polynomials

(V). Multiplication and division of polynomials
(A). To teach the multiplication and division of polynomials
(B). To use the parentheses in multiplication and division of polynomials

(VI). To teach the solution and uses of linear equations

(VII). Special products and factoring
(A). To teach the pupils how to take care of special products and factoring
(B). To teach the laws of exponents and how to use them

(VIII). To apply the four fundamental processes to fractions

(IX). To solve fractional and literal equations

(X). To illustrate the functional relation as expressed by words, formulas, and graphs

(XI). To give practice in dealing with roots and fractional exponents

(XII). To solve systems of linear equations in two unknowns

(XIII). To teach the uses of ratio, proportion, and variation

(XIV). Trigonometric ratios
(A). To teach the uses of trigonometric ratios
(B). To acquaint the pupils with the meaning
and usage of the terms tangent, sine, and cosine

(XV). To use various methods in solving quadratic equations


a. Objectives

(I). To give more emphasis to ideas and concepts
(II). To give simpler and more purposeful examples
(III). To bring the various parts of algebra more closely together by the dependence idea, or function concept, which is carried through the course by means of the formula, graph, table of values, and the equation
(IV). To develop factoring as a natural consequence of the fundamental processes of multiplication and division
(V). To well motivate the new parts of the work
(VI). To place special emphasis on the graph as a means of showing dependence

b. Specific objectives

(I). Symbols and graphs
   (A). To study the uses of symbols and graphs
   (B). To make and interpret various types of graphs

(II). The formulas
   (A). To study the uses of the formula
   (B). To develop the ability to write formulas
   (C). To know how to add, subtract, multiply,
and divide similar terms

(D). To evaluate and solve formulas

(E). To know what exponents are and how to use them

(III). Graphing formulas

(IV). Equations

(A). To know how to use the equation to solve problems

(V). Positive and negative numbers

(A). To use the positive and negative numbers

(B). To know how to use exponents

(VI). Dependence and problems

(A). To know the ways of indicating the dependence of one literal number on another

(B). To use axes of coordinates to locate points on cross-section paper

(VII). Fundamental processes with polynomials

(A). To apply the four fundamental processes to polynomials

(VIII). Special products and factoring

(A). To develop division and factoring through their direct relation to multiplication

(B). To emphasize the special products

\[(a + b)^2\], \[(a - b)^2\], and \[(a + b)(a - b)\]

(C). To use the simplest method for solving the quadratic equation (factoring)

(IX). Fractions

(A). To show the similarity between the
principles used in algebra in simplifying, multiplying, dividing, adding, and subtracting fractions, and the corresponding operations used in arithmetic

(B). To shorten algebraic operations by applying the principle that the changing of two of the three signs of a fraction does not change the value of the fraction

(X). Equations with fractions

(A). To solve problems by equations involving fractions

(B). To solve equations containing several literal numbers in terms of any one of these numbers

(C). To amplify the ideas of ratio and proportion

(XI). Linear equations

(A). To explain the different ways of solving a pair of linear equations

(XII). Radicals, powers, and roots

(A). To introduce the radical number and how to use it

(B). To solve equations involving radicals

(C). To find the square root of an arithmetical number

(D). To apply the final laws of exponents

(XIII). Quadratic equations
(A). To present four possible solutions of the quadratic equation

(B). To give methods for checking to see if the roots obtained by these four solutions of the quadratic equations are correct

(C). To give a solution and a check for simultaneous systems consisting of one linear equation and one quadratic

(D). To apply the quadratic equation to many practical problems

(XIV). Geometry and trigonometry

(A). To establish experimentally the three theorems of congruence of triangles

(B). To explain the three elementary theorems of similar triangles

(C). To define the tangent ratio and give a method for finding the values of a tangent

(D). To treat sine and cosine ratios in a manner similar to that of the tangent

(E). To show how to use a table of trigonometric ratios

(XV). Variation

(A). To present direct and inverse variation


a. General objectives

(I). To present algebra in such simple language
that the pupil can understand the explanations, and grow confident of his ability to learn from the text

(II). To stress the principle of the relationship between variables

(III). To center attention on the study of problems and of the formulas and equations to which these problems lead

(IV). To introduce processes dealing with parentheses, multiplication, factoring, fractions, and so forth when and because they are needed to solve specific types of problems

b. Specific objectives

(I). Literal numbers

(A). To know what is meant by literal numbers and how to use them

(II). Positive and negative numbers

(A). Know what is meant by positive and negative numbers

(B). Apply the four fundamental operations to positive and negative numbers

(C). Use the parentheses

(III). Linear equations

(A). Know the rules for solving linear equations and apply them

(IV). The solution of problems

(A). Know the preliminary steps in solving a
problem
(B). Solve problems involving equations and parentheses
(V). Solve equations containing fractions
(VI). Ratios and proportions
(A). To understand the meaning and uses of ratio and proportion
(B). To solve problems involving ratio and proportion
(VII). Make and interpret various kinds of graphs
(VIII). Sets of linear equations
(A). Use different methods to solve sets of linear equations
(B). Solve problems involving sets of linear equations
(IX). Formulas
(A). Make and use formulas
(B). Meaning and application of variables
(X). Multiplication and factoring
(A). Find the products and quotients of numbers containing exponents
(B). Multiply and factor polynomials
(C). Solve quadratic equations
(XI). Apply the four fundamental processes to fractions
(XII). Solve fractional equations
(XIII). Literal equations
(A). Solve literal equations
(B). Solve sets of literal equations
(XIV). Square roots
(A). Find square roots of numbers
(B). Apply the four fundamental operations to radicals

(XV). Quadratic equations
(A). Solve quadratic equations
(B). Use sines, cosines, and tangents in numerical trigonometry

9. Rothrock, David A., and Whitacre, Martha Anne., First Year Algebra

a. General objectives

(I). To lead the pupil to appreciate and interpret the formulas of geometry, physics, and other sciences through the essentials of algebraic notation and development

(II). To acquire skill in algebraic manipulation

(III). To give the pupil an insight into the many applications of mathematics to everyday life

(IV). To use verbal problems to stimulate the pupil's interest and to develop his confidence in his ability to analyze a situation and form his own equations

b. Specific aims

(I). Literal numbers and the formula

(A). To know what is meant by literal numbers and to know how to use them

(B). Get acquainted with the algebraic lan-
guage

(C). Know how to make and how to use the formula

(II). Positive and negative numbers

(A). Know what positive and negative numbers are

(B). Apply the four fundamentals to positive and negative numbers

(III). Addition and subtraction of algebraic numbers

(IV). Remove and insert parentheses

(V). Multiplication

(A). Multiply algebraic numbers caring for the:

(1). Signs

(2). Exponents

(B). Solve problems using multiplication

(VI). Division

(A). Divide algebraic numbers caring for the:

(1). Signs

(2). Exponents

(B). Solve problems using division

(VII). Solution of simple equations containing fractions

(VIII). Graphical representation

(A). Know the uses of graphs

(B). Make and interpret graphs

(IX). Special products and factoring
(A). To find special products
(B). To do factoring

(X). Solution of quadratic equations by factoring

(XI). Fractions
(A). To know what an algebraic fraction is
(B). Apply the four fundamental operations to fractions

(XII). Solution of fractional equations

(XIII). Solution of sets of linear equations

(XIV). Extraction of square root

(XV). Application of the four fundamental operations to radicals

(XVI). Quadratic equations
(A). To know what a quadratic equation is
(B). To solve quadratic equations by various methods

(XVII). Ratio, proportion, and variation
(A). Know what is meant by ratio, proportion, and variation
(B). Know how to use ratio, proportion, and variation

(XVIII). Numerical trigonometry
(A). Know the uses of:
   (1). Sine
   (2). Cosine
   (3). Tangent


a. General objectives
(I). To lead the pupil gradually into full insight of the logic of algebra
(II). To focus the teaching on the needs of the average pupil
(III). To effect mastery of the formula and equation
(IV). To test mastery of formula and equation by solving practical problems
(V). To introduce problem work to start with arithmetic solutions and gradually make the transition to algebraic solutions proper
(VI). To aid the acquisition of real power in problem solving:
   (A). By provision of interesting and practical problems
   (B). By unusually large amounts of drill supplied on the fundamental operations
   (C). Through the presentation of a definite technique for problem analysis
   (D). By regular measurement of progress in problem solving
   (E). By providing a program for the maintenance of skills
   (F). By teaching a mastery of the processes
   (G). By devices for arousing and maintaining motivation:
      (1). Competition with nation-wide groups of pupils by use of the problem
scales and Self-Testing Drills

(2). Competition with one's own past record

(3). Prevention of initial discouragement through leisurely attack and mastery of few processes

(4). Persistent use of the principle of "awareness of success or failure at the time of learning" (frequent standard tests)

(5). The variety and practicalness of the exercises and problems presented (common formulas, falling bodies, thermometers, etc.)

(H). To secure motivation through the interest aroused in mathematics itself

(I). To care for individual differences by exercises and problems graded for difficulty

(J). To call attention to the most frequent and persistent types of errors

(K). To present graphic methods

(L). To give attention to the history of algebra

(M). To make extensive and systematic use of primes, subscripts, and upper case (capitals) letters

(N). To provide generous illustrations

b. Specific objectives
(I). The nature and uses of algebra
   (A). To teach the nature and uses of algebra
   (B). To understand the algebraic vocabulary

(II). The equation
   (A). To know what an equation is and how to use it to solve problems

(III). Statistical graphs
   (A). To present various types of graphs
   (B). To graph formulas and linear equations
   (C). To solve problems by means of the graph

(IV). Positive and negative numbers
   (A). To apply the four fundamental operations to positive and negative numbers
   (B). To solve two equations by two operations

(V). Multiplication and division of monomials and polynomials
   (A). To give the rules of algebraic multiplication
   (B). To multiply monomials and polynomials
   (C). To give the rules of algebraic division
   (D). To divide monomials and polynomials
   (E). To remove parentheses

(VI). Equations of the first degree in one and two unknowns
   (A). To present equations of the first degree in one unknown
   (B). To use fractional coefficients in equations
   (C). To know what a literal equation is and
how to use it
(D). To solve motion and mixture problems
(E). To solve simultaneous equations by the
graphic, addition or subtraction, and
substitution method

(VII). Special products and factoring
(A). To understand what special products and
factoring are and how to do them
(B). To solve equations by factoring

(VIII). Fractions. Fractional and literal equations
(A). To know what an algebraic fraction is
(B). To apply the four fundamental operations
to fractions

(IX). Ratio and proportion. Variation and dependence. The function concept. Introduction to
trigonometry
(A). To know the meaning of ratio, proportion,
variation and dependence
(B). To use ratios and proportions in solving
problems
(C). To define and use direct and inverse
variation
(D). To know what is meant by similarity and
how to use this knowledge
(E). To know how to measure directly and in-
directly
(F). To know what is meant by trigonometric
ratios
(G). To know what is meant by tangent, sine,
cosine, and interpolation

(H). To solve problems dealing with trigonometric functions

(X). Roots and powers. Equations of the second degree

(A). To use various methods to find roots and powers

(B). To use different methods to solve quadratic equations

Il. Schorling, Raleigh, and Clark, John R., Schorling—Clark

Modern Algebra: Ninth School Year

a. Specific objectives

(I). How to express the relation between numbers that change together

(A). To use graphs, tables, and formulas to express relations between numbers

(B). To use letters for numbers

(C). To practice with formulas

(D). To graph formulas

(II). Using the formula; the axioms; the use of equations in problem solving

(A). To show different forms of the same formula

(B). To learn what the four great principles or laws used in solving equations are

(C). To learn what an equation is

(D). To use the axioms

(E). To use equations to solve problems

(III). How to apply important relations of the
right triangle

(A). To introduce the word "tangent"
(B). How to find the tangent of an angle
(C). How to use the tangent to solve practical measurement problems
(D). To use the cosine and sine ratios to solve problems
(E). To use the tangent, cosine, and sine table

(IV). The use of positive and negative numbers
(A). To learn the meaning of positive and negative numbers
(B). To apply the four fundamental operations to positive and negative numbers
(C). To learn the important law of exponents in division

(V). Learning to apply the four fundamental operations to algebraic numbers
(A). To learn some useful names in algebra
(B). Addition of like terms
(C). Addition of polynomials
(D). Subtraction of algebraic signed numbers
(E). Multiplying a polynomial by a monomial
(F). Multiplication of a polynomial by a polynomial
(G). Division of a polynomial by a monomial
(H). Division of a polynomial by a polynomial

(VI). Products and factors
(A). Translating algebraic language into
(B). To find special products by the use of:

1. The square of the sum of two numbers
2. The square of the difference of two numbers
3. The product of the sum and difference of two numbers

(C). Factoring algebraic expressions

(D). Finding prime factors

(VII). More practice in solving equations

(A). To solve easy equations containing negative numbers

(B). To solve equations containing parentheses

(C). To solve problems expressed in words

(D). To solve various kinds of problems

(VIII). Some important principles of arithmetic extended to algebraic fractions

(A). To know what an algebraic fraction means

(B). To reduce fractions to lowest terms

(C). To apply the four fundamental operations to fractions

(IX). Literal and fractional equations

(A). To solve equations that contain fractions with binomial denominators

(B). To construct formulas

(X). Simultaneous linear equations
(A). To graph and study linear equations
(B). To solve simultaneous equations by:
(1). The graphic method
(2). The addition or subtraction method
(3). The substitution method

(XI). How numbers change together; variation and proportion
(A). To solve exercises containing variables and constants
(B). To understand direct variation
(C). To know the different forms of proportion
(D). To solve problems involving proportions
(E). To graph direct variation
(F). To understand inverse variation
(G). Applying the law of levers

(XII). Roots and powers
(A). To teach how to find square roots
(B). To use radicals
(C). To use fractional exponents

(XIII). How to solve equations of the second degree
(A). To solve quadratic equations:
(1). Graphically
(2). By completing the square
(3). By factoring

(XIV). What every one should know about statistics
(A). To find the average, median, and mode
(B). To arrange statistical data in a fre-
Logarithms and approximate numbers

(A). Purpose of logarithms

(B). To build and use a table of powers of 10

(C). To use the table of logarithms

(D). Using logarithms to find powers

Use of the slide rule

The measurement of angles

12. Stone, John C., *The New Mathematics First Year Algebra*

a. General objectives

(I). To develop the power to represent quantitative relationships by formulas and equations

(II). To develop the power to interpret quantitative relationships by formulas and equations

(III). To develop the skills needed in the computation which is required in using formulas and equations

(IV). To make skills in algebra more permanent by frequent recall throughout a long period of time

(V). To perform all the fundamental processes with algebraic numbers accurately and quickly, and thus to evaluate and solve all types of formulas in order to solve all kinds of equations in the first and second degree

(VI). To express all kinds of quantitative relationships either as formulas or equations
b. Specific objectives

(I). The language of algebra

(A). To get acquainted with the language of algebra

(B). To learn the meaning and use of general numbers

(C). To make and evaluate formulas

(D). To make and interpret graphs

(E). To translate the statements of problems into equations and to solve them

(II). A new kind of number

(A). To learn the meaning and use of signed numbers

(B). To add and subtract signed numbers

(C). To apply knowledge already learned to problem solving

(III). Multiplication and division of algebraic numbers

(IV). Special products and factors

(A). To use special products and factors

(B). To solve equations by factoring

(V). Fractions

(A). To know what an algebraic fraction is and how to reduce it to lowest terms

(B). To care for the signs of a fraction

(C). To change algebraic fractions to the lowest common denominator

(D). To multiply and divide algebraic frac-
(E). To change fractions to mixed expressions
(F). To change a mixed expression to a fraction
(G). To change a fraction to a given denominator
(H). To add and subtract fractions
(I). To handle complex fractions
(J). To be able to solve literal equations
(VI). Solving equations having two unknown numbers
(A). Solve two linear equations by the graphic method
(B). Solve simultaneous equations without using graphs
(VII). Powers, roots, radicals, and quadratic equations
(A). To understand and to be able to use powers, roots, and radicals
(B). Know what a quadratic equation is and how to solve it
(C). To make and use a formula for solving quadratics
(D). To solve problems involving quadratics
(E). To graph quadratic equations
(VIII). Ratio, variation, and proportion, and numerical trigonometry
(A). Know the meaning of ratio, variation, and proportion
(B). To use trigonometric ratios
13. Thorndike, Edward Lee, *The Thorndike Algebra*

a. Specific objectives

(I). Formulas and equations

(A). To use formulas

(B). To use parentheses

(C). To use the equation to solve exercises and problems

(II). Algebraic numbers

(A). To apply the four fundamental operations to algebraic numbers

(B). To use positive and negative numbers

(C). To apply the four fundamental operations to positive and negative numbers

(III). Algebraic expressions and formulas: Addition and subtraction

(A). To get acquainted with the algebraic language

(B). To add and subtract algebraic expressions

(C). To use the formula

(IV). Multiplication and division: Products and factors

(A). To find algebraic products

(B). To multiply polynomials by monomials

(C). To multiply polynomials by a polynomial

(D). To know the simple uses of powers and roots
(E). To divide a polynomial by a monomial
(F). To divide a polynomial by a polynomial
(G). To simplify expressions
(H). To do factoring
(I). To find square roots

(V). Fractions: Fractional equations and formulas
(A). To apply the four fundamental operations to fractions
(B). To solve fractional equations
(C). To derive and transform formulas

(VI). Powers and roots
(A). To find square and cube roots
(B). To apply the four fundamental operations to roots
(C). To solve equations containing powers
(D). To solve problems involving roots

(VII). Relations expressed by tables, graphs, and formulas
(A). To understand tables and graphs
(B). To make tables and graphs
(C). To read and interpret graphs
(D). To use tables and graphs as a means of expressing the relation between one variable and another

(VIII). Equations expressing important mathematical relations: Direct and inverse variation: Equations of straight lines: Sets of equations
(A). To solve simultaneous linear equations
(B). To find the equation of any straight-line graph
(C). To understand direct and inverse variation

(IX). Solving quadratic equations
(A). To solve any quadratic equation in one unknown
(B). To solve problems by quadratic equations

(X). Ratio and proportion
(A). To know the use of the words ratio and proportion
(B). To compute with ratios

(XI). Tangents, sines, and cosines
(A). To understand the meaning of the tangent ratio and the uses of it
(B). To understand the meaning of sine ratio
(C). To understand the meaning of cosine ratio

(XII). The general principles of algebra
(A). To use directed numbers
(B). To teach the extension of symbolism
(C). To solve numerical equations
(D). To factor and simplify expressions

(XIII). Logarithms and other labor saving devices
(A). To know the nature and uses of logarithms

a. General objectives

(I). To present algebra so it can be clearly understood by boys and girls

(II). To apply fundamental principles rather than to manipulate symbols

(III). To present a thinking algebra

(IV). To make algebra an interesting and meaningful subject

(V). To study relationships between quantities as expressed by the formula, the equation, and the graph

(VI). To show the dependence of one quantity on another

(VII). To bring out the idea of a mathematical law

(VIII). To give a more complete treatment of the formula

(IX). To learn the solution of equations based entirely upon a few fundamental principles

(X). To use the graph freely as a means of picturing the relationships expressed by formulas and equations

(XI). To make numerical trigonometry meaningful to the pupils

(XII). To effect complete mastery of each topic

(XIII). To provide an abundance of interesting problems to afford a challenge to the pupil's initiative
b. Specific objectives

(I). Formulas
   (A). To know the meaning of the formula
   (B). To know how to make and use the formula

(II). Formulation and solution of equations

(III). Positive and negative numbers
   (A). Apply the four fundamental operations to signed numbers
   (B). Learn the language of algebra

(IV). Fundamental operations
   (A). Apply the four fundamentals to monomials and polynomials
   (B). Know and use the laws of exponents in division and multiplication
   (C). Remove parentheses

(V). Special products and factoring
   (A). Find special products and do factoring
   (B). Find square roots

(VI). Linear equations
   (A). Learn the language of equations
   (B). Solve linear equations
   (C). Solve literal equations
   (D). Make new formulas from old ones

(VII). Fractions
   (A). Know the meaning of a fraction
   (B). Apply the four fundamental operations to fractions
   (C). Solve and evaluate formulas

(VIII). Fractional equations
(A). To find the lowest common denominator
(B). To solve equations with fractional coefficients
(C). To solve fractional equations
(D). To solve equations containing decimals
(E). To solve literal equations
(F). To solve and evaluate formulas containing fractions
(G). To solve problems containing fractional equations

(IX). Sets of linear equations
(A). Graphing sets of linear equations
(B). Solving sets of linear equations by various methods
(C). Solving sets of literal equations
(D). Solving problems dealing with sets of linear equations

(X). Ratio, proportion, and variation
(A). Know the meaning of ratio, proportion, direct and inverse variation
(B). Know how to use ratio, proportion, direct and inverse variation

(XI). Numerical trigonometry
(A). Know the meaning of sine, cosine, and tangent
(B). Know how to use the sine, cosine, and the tangent in solving problems dealing with measurements
(XII). Powers and roots
   (A). Know the laws of exponents and how to apply them
   (B). Find squares and roots from a table
   (C). Use the radical to solve exercises

(XIII). Quadratic equations
   (A). To solve quadratic equations by:
      (1). Completing the square
      (2). Using the quadratic formula
   (B). To solve problems containing quadratic equations
   (C). To solve quadratic equations graphically

C. Objectives in First Year Algebra
   from Courses of Study

1. Athens, Ohio
   a. Objectives
      (I). General
         (A). To introduce training for those wishing to pursue higher mathematics and to enter the professions in which a large amount of mathematical skill is necessary
         (B). To develop accuracy
         (C). To develop concentration
         (D). To improve quality of thinking
      (II). Specific
To master to a workable degree the following subjects: Addition, subtraction, multiplication, division, simple and simultaneous equations, graphs, special products, factoring, formulas, fractions, and square roots

To produce quadratic equations and radicals

2. Cleveland, Ohio

a. Aims

"The primary purposes of the teaching of mathematics should be to develop those powers of understanding and analyzing relations of quantity and of space which are necessary to a better understanding of life and the universe about us, and to develop those habits of thinking which will make these powers effective in the life of the individual." ¹

(I). Practical aims

(A). To acquire command of mathematical principles and processes necessary to most people regardless of vocation

(B). To acquire command of mathematical principles and processes needed for special professions or vocations

(C). To acquire command of mathematical principles and processes required in other sciences

(II). General aims

(A). To develop generally used concepts of number and space relations and certain mathematical thought modes

(B). To improve efficiency which may be transferred to other closely related fields

(III). Cultural aims

(A). To develop ideals of precision in statement and thought

(B). To develop ideals of perfection as to logical structure and reasoning

(C). To develop discrimination between true and false

(D). To develop appreciation of the role of mathematics in the development of civilization

(E). To develop enjoyment of precision, of logical reasoning, and satisfaction in the recognition of truth

b. Objectives for first half of ninth year

(I). To have pupils understand and use in thinking about the common quantitative problems of school, business workshop, and the sciences the following:

(A). Literal numbers
(1). Uses
(2). Formulas
(3). Equations

(B). Directed numbers

(C). Ratio and proportion

(D). Graphs

(II). To have pupils acquire the understanding and manipulative skill in facile and accurate quantitative thinking by:

(A). Reading and writing simple algebraic expressions including:

(1). Monomials
(2). Polynomials
(3). Signed numbers
(4). Exponents
(5). Quantities with parentheses

(B). Performing the four fundamental operations on:

(1). Monomials
(2). Polynomials
(3). Fractions

(C). Substituting a value for a letter in a formula or other equation

(D). Expressing simple mathematical ideas or rules in symbols

(E). Translating formulas into rules

(F). Checking work by repetition, reverse process, or substitution

(G). The meaning and use of algebraic expres-
(H). Factoring simple algebraic expressions

(III). To have pupils acquire an understanding and appreciation of the development of certain usages of algebra as follows:

(A). The origin of the symbols of operation

(B). The development of certain ideas, as the concept of zero

(C). The development of the use of positive and negative numbers

(IV). To have pupils practice neatness and accuracy and to take pride in doing so, using such devices as follows:

(A). Setting all work in an orderly and systematic manner

(B). Checking all work before considering it finished

(V). To have pupils accept cheerfully the challenge inherent in the subject of mathematics and to use persistently their fullest intellectual powers in carrying on the work to a successful conclusion

c. Objectives for the second half of the ninth year

(I). To have pupils understand and use in thinking about the common quantitative problems of school, business, work shop, and the sciences the following:

(A). Literal numbers—formulas
(II). To have pupils acquire the understanding and manipulative skill useful in facile and accurate quantitative thinking by:

(A). Reading and writing algebraic expressions

(B). Performing the four fundamental operations on

(1). Monomials
(2). Binomials and other polynomials
(3). Fractions
(4). Radicals

(C). Substituting a value for a letter in a formula or expression

(D). Expressing simple mathematical ideas or rules in symbols

(E). Translating formulas into rules

(F). Checking work by repetition, reverse process, or substitution

(G). The powers and roots of algebraic expressions including signed numbers

(H). Factoring simple algebraic expressions

(III). To have pupils acquire an understanding and appreciation of the development of certain usages of algebra as follows:

(A). The origin of the symbols of operation
(B). The development of certain ideas in mathematics such as the concept of zero

(C). The development of the use of positive and negative numbers

(IV). To have pupils understand and use algebraic concepts or modes of thought as distinguished from algebraic techniques, in reading, thinking, and talking about the ordinary things in which educated people are interested

(V). To have pupils practice neatness and accuracy and to take pride in doing so

(VI). To have pupils accept cheerfully the challenge inherent in the subject of mathematics and to use persistently their fullest intellectual powers in carrying on the work to a successful conclusion

3. Fort Wayne, Indiana

a. Purposes

(I). To give pupils keener understanding of the abstract workings of mathematical principles

(II). To give the algebra necessary to simplify solutions of arithmetical problems

(III). To enable pupils to read, grasp, and construct literal formulae and graphs

(IV). To develop the power to see relationships

(V). To give training in thinking in logical
4. Lorain, Ohio
a. Aims

(I). To develop the power to interpret and use symbolic formulas
(II). To round out the work of arithmetic by generalizing and extending its processes
(III). To develop the equation and apply it to the solution of a great variety of problems
(IV). To develop the concept of a function and its graphic representation
(V). To furnish such material and develop such skill as may be needed in the later study of mathematics and the various sciences
(VI). In general, to assist pupils to apply known principles to the solution of new problems

5. Milwaukee, Wisconsin
a. General aims

(I). To increase the understanding and appreciation of the formula—its construction and use
(II). To develop the ability to construct and to interpret correctly graphs of various kinds
(III). To extend the pupil's power to work out problems by setting up and solving the necessary equations
(IV). To develop the fundamental laws of algebra and to give the necessary drill in algebraic technique
b. Practical aims

(I). To provide a tool to be used in the study of various branches of learning in which some knowledge of mathematics is necessary

(II). To provide the mathematical knowledge and the skill in handling algebraic and trigonometric formulas and expressions and geometric forms which will assist workmen in various kinds of work to efficiency in their work and to an understanding of, and interest in the things they are doing by giving some degree of life to the "cut and dried" formulas and tables that they are constantly using

(III). To provide the mathematical knowledge necessary for the understanding of ordinary magazine reading and conversation with which an intelligent citizen comes in contact very frequently

c. Mental discipline

(I). The habit of thinking a mathematical proposition through and, with given data arriving at a logical conclusion

(II). The power to analyze and come to the truth in a given mathematical situation

(III). Neatness, accuracy, and conciseness in both written and spoken language

d. Cultural values

(I). To encourage the student
(A). To correctly estimate values
(B). To gain the sense of mastery
(C). To get from life the most enjoyment possible

6. San Francisco, California

a. General objectives for grade low nine

(I). Increased facility in:
(A). The uses of literal and signed numbers
(B). Manipulating formulas
(C). Manipulating simple equations
(D). Solving problems
(E). Performing fundamental operations with the enlarged number system

(II). Increased knowledge of special products and skill in factoring

b. Specific objectives for grade low nine

(I). Literal numbers, negative numbers, formulas
(A). Knowledge of meaning of common algebraic terms and symbols
(B). Understanding of negative numbers
(C). Ability to read and use simple formulas

(II). Addition and subtraction
(A). Ability to add and subtract signed numbers and polynomials
(B). Ability to simplify and evaluate algebraic expressions
(C). Skill in checking results obtained from addition or subtraction

(III). Simple equations and problems
(A). Ability to solve and check simple equations
(B). Knowledge of use of axioms in solutions
(C). Greater facility in manipulation of formulas
(D). Skill in use of equation as a ready tool for solving problems
(E). Appreciation of utility of algebra

(IV). Multiplication and division
(A). Facility in multiplying and dividing signed numbers and polynomials
(B). Understanding of first two laws of exponents

(V). More complicated equations and problems
(A). Added skill in removing parentheses
(B). Increased ability in solving linear equations in one unknown
(C). Facility in solving practical problems

(VI). Special products and factoring
(A). Understanding of products and factors
(B). Ability to get mental products of two binomials
(C). Facility in recognizing certain products and giving rapidly the factors
(D). Ability in solving equations by factoring and simple problems involving quadratic equations which can be factored

C. General objectives for grade high nine
(I). Skill in:
   (A). Performing fundamental operations with fractions
   (B). Solving fractional equations
   (C). Solving quadratic equations
   (D). Solving simultaneous linear equations

(II). Facility in:
   (A). Use of roots and powers
   (B). Development of further power to solve practical problems

Specific objectives for grade high nine

(I). Fractions and fractional equations
   (A). Skill in performing fundamental operations with fractions
   (B). Appreciation of utility of factoring in obtaining least common multiple
   (C). Fixation of laws of signs
   (D). Further skill in manipulating formulas
   (E). Ability to solve fractional equations and practical problems

(II). Simultaneous linear equations
   (A). Appreciation of graph as picture equation
   (B). Accuracy and speed in solving simultaneous linear equations by graphing or elimination
   (C). Ability to solve problems involving linear equations

(III). Roots, powers, and exponents
(A). Knowledge of meaning of roots, powers, indices, exponents, radicals
(B). Skill in finding exact square roots by table, graph and division
(C). Ability to change fractional exponents to radicals, and vice versa
(D). Facility in reducing radicals and in performing fundamental operations with them

(IV). Quadratic equations
(A). Skill in solving quadratic equations by different methods
(B). Ability to solve problems involving such equations
(C). Facility in judging best method to use
(D). Knowledge that problem-solving is final aim of instruction in algebra

7. Trinidad, Colorado
a. Practical aims
(I). To develop the ability to grasp, interpret, and master the simple problem situations which are of common occurrence in everyday life

(II). To stimulate progressive increase in understanding of fundamental processes, and to develop skill and precision in using them

(III). To train in the exercise of common sense and judgment in computing, and to develop self-
reliance in the use of the check
(IV). To cultivate a definite degree of skill
in using the ordinary tools of problem
solving—the formula, the graph, and the
equation—together with the essential
technique
(V). To develop the ability for mathematical
reading
(VI). To provide the necessary foundation for
understanding higher mathematics and
science

b. Disciplinary and cultural aims
(I). To acquire the ability to think in abstract
terms, and to concentrate on the processes
of mathematical combination as distin-
guished from particular numbers
(II). To furnish material for sustained mental
effort and to require such sustained
effort
(III). To foster habits of accuracy, neatness, and
order
(IV). To develop appreciation of logical continu-
ity of thought
(V). To give an insight into the processes that
are used as the basis for the great con-
structive work of the world in astronomy,
navigation, and the physical sciences

8. Florida

a. Objectives for mathematics
(I). To learn mathematics that is closely related to life
(II). To compute quickly and accurately
(III). To think independently
(IV). To apply mathematical knowledge to their everyday life

9. Idaho
a. Primary purposes

(I). To develop powers of understanding and of analyzing relations of quantity and of space which are necessary to an insight into and control over our environment and to an appreciation of the progress of civilization in its various aspects
(II). To develop those habits of thought and of action which will make these powers effective in the life of the individual

10. Indiana
a. General objectives in secondary mathematics

"These objectives have to meet the needs of three groups of students: first, those who will not pursue the subject beyond the ninth year, either because they will not continue in high school, or because they have little aptitude for the subject; second, those who will pursue the study further because of their natural aptitude and love for the subject; and third, those who will require a maximum course so that they may follow work
along their special vocational lines."

(I). Objectives for the first group

(A). To increase the ability to perform the ordinary mathematical computations with a reasonable degree of accuracy and speed

(B). To develop the ability to read and interpret such statistical and graphical data as occur in popular discussions of current scientific, social, industrial, and civic problems

(C). To afford that socializing influence which results from an understanding and appreciation of the dependence of everything on something else

(D). To increase the ability to analyze a situation, and to appreciate its quantitative aspects

(E). To increase the tendency toward accuracy in estimating and checking results

(F). To develop the ability to do accurate quantitative thinking

(G). To provide the practical knowledge of mathematics that is necessary to meet the immediate needs of pupils, and,

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2Revised Tentative Course of Study in Mathematics for Secondary Schools in Indiana, Bulletin No. 100D. Rev., Department of Public Instruction (1930), p. 10.
at the same time, to serve as a basis for preparation for vocations requiring additional training in mathematics. (H). To teach thrift by presenting challenging financial problems and projects. (I). To increase the ability to understand and use mathematics that those who appreciate it will continue the study for the love of the subject. (II). Objectives for the second group: (A). To continue to develop skill in the fundamental processes. (B). To appreciate the role mathematics has played in the progress of civilization. (C). To stimulate an interest in the historical and recreational side of mathematics. (III). Objectives for the third group: (A). To increase the ability to collect, organize, and chart data for scientific and practical investigations. (B). To further develop the function concept. (C). To provide for the vocational needs of the pupils who plan to specialize along such lines as engineering, science, and architecture. (D). To give some understanding of the common phases of mathematical investments,
such as life insurance, annuities, savings accounts, and statistics.

b. General objectives for ninth year mathematics

(I). To increase the ability to think accurately along mathematical lines
(A). Through derivations, evaluations, and use of formulas
(B). Through the use of the equation as a problem-solving device
(C). Through the development of the ability to read and interpret graphical representations
(D). Through the development of the ability to use directed numbers
(E). Through the acquisition of an algebraic vocabulary

(II). To develop the practice of checking all work so that the pupil may experience the satisfaction that comes from confidence in the result and mastery of the subject

(III). To develop a deeper insight into mathematical situations encountered in arithmetic which admit of better treatment in algebra

c. Specific objectives for first semester algebra

(I). Literal numbers
(A). To bridge the gap between grades eight and nine by review of the use of literal numbers
(B). To develop the ability to understand
and use algebraic words and phrases

(II). Directed numbers (positive and negative)

(A). To develop the ability to use directed numbers

(1). Using familiar examples to develop the idea of directed numbers

(2). Adding directed numbers

(3). Multiplying directed numbers

(4). Dividing directed numbers

(5). Subtracting directed numbers

(III). Linear equations and parentheses

(A). To acquire a knowledge of the principles involved in the solution of linear equations

(1). Solving linear equations by division

(2). Solving linear equations by addition

(3). Solving and checking, using transposition

(B). To acquire a knowledge of the symbols of grouping and the principles involved in removing them as an aid in solving equations

(C). To learn that a formula is a rule written in algebraic shorthand

(D). Making formulas and solving problems involving the use of the formula
(IV). Solution of verbal problems
   (A). To learn to translate verbal statements into equations
   (B). To increase the ability to solve equations
      (1). Forming and solving equations
      (2). Forming and solving equations containing fractions

(V). Ratio and proportion as an aid in solving problems

(VI). Graphs
   (A). To learn to make and interpret graphs
   (B). To acquire some familiarity with the vocabulary pertaining to graphing
   (C). Locating points and graphing linear equations

(VII). Sets of linear equations
   (A). To learn the methods of solution of linear equations having two unknowns
      (1). Solving sets of linear equations having two unknowns, by the use of the graph
      (2). Solving sets of linear equations having two unknowns by the multiplication addition method
      (3). Solving sets of linear equations having two unknowns by the substitution method
      (4). Verbal problems which involve the
forming and solving of sets of linear equations having two unknowns

(5). Checking every problem by substituting the values of the unknowns in both equations

(6). In the case of verbal problems, checking by the conditions stated in the words of the problem, not by substitution in the equation only

11. Kentucky

a. General aims and objectives

(I). To satisfy the needs of the various types of pupils who take the subject

(II). To exhibit mathematical type of thought

(III). To discover mathematical abilities

(IV). To help to a better understanding of the laws of nature

(V). To give sufficient skill in the actual performance of mathematical processes to meet the future needs of the pupil

(VI). To show the mathematical relationships that exist in the social organism and in the activities of modern life

b. Specific aims and objectives

(I). To acquaint the pupil with:

(A). The use of letters to represent numbers
(B). The use of the equation

(C). The construction and evaluation of formulae

(D). Methods of graphical representation

(E). The properties of the more important spaces, forms in the expression, and determination of relationships

(II). To give the pupil some idea of the use of geometry and trigonometry of the right triangle

(III). To maintain and increase his skill in arithmetic and give him a more thorough understanding of some of its rules

(IV). To furnish the pupil with an appreciative basis for other subjects involving the use of algebra

12. New York

a. Objectives for ninth year of four-year high schools

(I). The ultimate goal of mathematical training

(A). The development of power to solve problems of quantity and space (of appropriate difficulty) with reasonable skill and confidence

(II). The primary purposes of the teaching of mathematics

(A). To develop those powers of understanding and of analyzing relations of quantity and of space which are necessary to an
insight into and control over environment and to an appreciation of the progress of civilization in its various aspects

(B). To develop those habits of thought and of action which will make these powers effective in the life of the individual

(III). Underlying principle

(A). The idea of relationship between variables, including the methods of determining and expressing such relationships

13. New York

a. Aims for first year algebra in junior high schools

(I). Preparation for further study

(II). Command of fundamentals

(III). Ability to read and construct formulae

(IV). Ability to read and construct graphs

(V). Mental efficiency

(VI). Appreciation of algebra, its general meaning, and its place as an interesting, useful, rational subject

(VII). Desire for higher mathematics

(VIII). Formation of habits of accuracy, concentration, common sense, judgment

(IX). Applications of algebra in daily life

(X). Relation of algebra to other subjects
(XI). Benefit of algebra to mankind
(XII). Accuracy and clearness in thinking
(XIII). Ability to think in quantitative terms with
a high degree of accuracy when dealing with
familiar concrete material
(XIV). Logical thinking
(XV). Power to think straight
(XVI). Understanding of the essentials of mathematics
necessary to intelligent citizenship

14. Oregon

a. Objectives

(I). To review, develop, and extend the most im-
portant processes of arithmetic
(II). To develop the equation as the central topic
of algebra
(III). To develop the use of graphs and graphic rep-
resentations in general
(IV). To develop the ability to analyze problems,
to formulate them, and to interpret the
results obtained
(V). To develop those powers of understanding and
analyzing relations of quantity which are
needed to give a better understanding and
appreciation of life

15. Wisconsin

a. Objective

(I). To develop those powers of understanding and
of analyzing relations of quantity and of
space which are necessary to an insight into and control over our environment and to an appreciation of the progress of civilization in its various aspects (II). To develop those habits of thought and of action which will make these powers effective in the life of the individual

D. Results of the Survey of General Objectives in First Year Algebra

1. The results of the survey showed that practically all the authorities agree that first year algebra has for its general objectives:
   a. An appreciation of the uses and values of algebra
   b. A provision for a knowledge of the language of algebra
   c. An encouragement of the development of the powers to think and to do
   d. A provision for the practical needs of the child
   e. An encouragement of the development of skill in reading, analyzing, and solving algebraic exercises and problems

E. Results of the Survey of Specific Objectives in First Year Algebra

The methods books and courses of study do not discuss the specific objectives as elaborately as they discuss the general
objectives of first year algebra

The textbooks are agreed on the topics on which the specific objectives of first year algebra are based. They differ, somewhat, as to the sequence of those topics.

1. The authorities who discuss specific objectives agree that first year algebra has for its specific objectives:

a. A provision for the learning of the symbols of algebra

b. A provision for the learning of the concepts of algebra

c. An encouragement of the development of the powers to read, analyze, and take care of the phases of algebra as listed in Table II

F. Explanation of Tables I and II

Table I is based on a study of eleven methods books, fourteen textbooks, and fifteen courses of study. Table I may be read as follows: The objectives, powers, and attitudes are used by nine out of the eleven methods books, ten out of the fourteen textbooks, and eleven out of the fifteen courses of study, used to make this study.

Table II is based on the same sources on which Table I is based. The same procedure may be used for reading Table II that was used in reading Table I
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<thead>
<tr>
<th>Objectives</th>
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TABLE II
A SUMMARY OF RESULTS OF THE SURVEY OF TOPICS UPON WHICH THE SPECIFIC OBJECTIVES OF FIRST YEAR ALGEBRA ARE BASED

<table>
<thead>
<tr>
<th>Topics upon Which the Specific Objectives Are Based</th>
<th>Sources</th>
<th>Methods</th>
<th>Text-Books</th>
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CHAPTER III

I. OBJECTIVES OF FIRST YEAR ALGEBRA

A. General Objectives

A list of objectives has an invaluable part to play in the construction of a test. The following objectives were utilized in making the test.

1. First year algebra should provide for:
   a. A knowledge of the uses and values of algebra
   b. A knowledge of the language of algebra
   c. The development of the powers to think and do
   d. The practical needs of the child
   e. The development of skill in reading, analyzing, and solving algebraic exercises and problems

B. Specific Objectives

1. First year algebra should provide for the ability to:
   a. Use letters in algebra
   b. Use formulas
      (I). Translate verbal expressions into formulas
      (II). Evaluate formulas
      (III). Work with ordinary simple formulas
   c. Graph
      (I). Represent points by means of the usual coordinate system
(II). Read values from a graph
d. Use equations
   (I). Translate a verbal problem into an equation
   (II). Translate an equation into words
   (III). Solve linear equations with one unknown
   (IV). Solve linear equations containing common
        or decimal fractions
   (V). Use equations in solving problems
e. Understand directed numbers
   (I). Use directed numbers practically
   (II). Apply the four fundamental operations to
        directed numbers
   (III). Use directed numbers in graphs
   (IV). Use directed numbers in formulas
   (V). Remove one set of parentheses
f. Perform operations with polynomials
   (I). Apply the four fundamental operations to
        polynomials
   (II). Factor
   (III). Find prime factors
   (IV). Apply the laws of exponents
g. Understand and use algebraic fractions
   (I). Understand the principle of signs
   (II). Reduce a fraction to lowest terms
   (III). Apply the four fundamental operations to
        fractions
h. Understand and use fractional equations
   (I). Clear an equation of fractions
(II). Solve fractional equations
i. Understand what is meant by ratio, proportion, and variation
(II). Solve problems in variation
j. Solve sets of linear equations
   (I). Solve by the graphic method
   (II). Solve by the substitution method
   (III). Solve by the multiplication–addition method
(IV). Solve problems involving sets of linear equations
k. The development of:
   (I). Speed
   (II). Skill
   (III). Accuracy
   (IV). Clearness
   (V). Logical thinking
   (VI). Self–reliance
CHAPTER IV

I. THE TECHNIQUES USED IN CONSTRUCTING THE TEST

A. Preliminary Test

In constructing a test it is necessary to follow some order of procedure.

Chapter IV presents the techniques used in constructing the test.

1. Building the Test. In constructing the preliminary test, Ruch's general order of procedure was followed.

   a. Drawing up a "Table of Specifications." "The term "Table of Specifications" was adopted for the sake of emphasizing the need for a general guide or skeleton in building a test. Such a table guards against the omission of essential items, the over-emphasis of minor topics, and improper balance of the sampling." 2 The "Table of Specifications" goes a considerable distance in establishing the validity of the final test when completed. The "Table of Specifications" which was drawn up is shown as Table III and Table IV.

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1G. M. Ruch, The Objective or New-Type Examination (Chicago: Scott Foresman and Co., 1929), p. 149.
2Ibid., p. 149.
b. Drafting the items in preliminary form with the "Table of Specifications" at hand, the next step was that of writing down tentative test items. The topics and sub-topics were considered in turn, and items covering the high points were listed. Two objectives were kept in mind: 
(I). To cover the field thoroughly but at the same time to avoid trivial points; and 
(II). To decide which objective technique or type (true-false, completion, multiple-choice, or matching) was best suited to handling the particular question in mind. 

Approximately 300 preliminary items were selected for the test.

c. Deciding upon the length. The basis for deciding upon the length of the preliminary test were: 
(I). The test must be long enough that no student can complete it in ninety minutes.3 
(II). The number of pages of the text to be covered, 242 pages for Form A, and 252 pages for Form B.4 
(III). Approximate number of worthwhile items which could be made. The number of items was reduced to 126. Two forms of the test were made, sixty-three items to each form.

3 See Appendix A
4 Ibid A
d. Editing and selecting the final items. In the process of editing and selecting the final items the following points were kept in mind:

(I). Avoidance of misleading statements
(II). Clear sentence structure
(III). Simple language
(IV). Careful punctuation
(V). Good form for the algebraic expressions
(VI). The objectives of first-year algebra

e. Rating the items for difficulty. The items were grouped into ten groups. The items were rated in each group according to difficulty. The groups were as follows:

(I). Linear equations
(II). Addition, subtraction, multiplication, and division of polynomials
(III). Simplification
(IV). Changing verbal expressions to algebraic expressions
(V). Evaluating formulas
(VI). Factoring
(VII). Multiple choice based on algebraic language
(VIII). Sets of linear equations
(IX). Written problems
(X). Graphs

Some desirable reasons for this grouping were as follows:

(I). The material seemed to lend itself to this grouping
(II). "Encouragement is offered to all pupils when they find something easy on which to begin

(III). The slow pupil is not penalized until he approaches the end of the first group

(IV). The good pupil is checked as he approaches the first "peak" of difficulty, but he increases his speed after passing this point.5

(V). Easier to rank the items according to difficulty within the group

(VI). Easier to evaluate the items

A classification of the various types of problems in Nyberg's text led to the above grouping.

f. Breaking the items into equivalent forms. When the items were being formulated an effort was made to select them in pairs. Two similar items were selected for each pair. One of the items in each pair was placed in Form A, and the other item was placed in Form B. A few exceptions were necessary in order to take care of the additional ten pages of material that Form B was to cover.

g. Rearranging the items in order of difficulty. This step was largely taken care of in the first arrangement of items in each group according to

5John E. Houston, A Standardized Test for the First Semester of World History (Masters Thesis Indiana State Teachers College, 1931), p. 15.
difficulty

h. Preparing the instructions for the test. The instructions were made simple, clear, and brief. Examples were given where they were needed in order to give a better idea of what was to be done.

i. Making the answer key. A suitable key was made for scoring the test, the writer keeping in mind the elements of efficiency and economy relative to time and effort. The scoring of the test was facilitated by arranging the items so that all the results would by placed on the right side of each page.

j. Deciding upon rules for scoring. Rules for scoring the preliminary test were as follows:
   (I). Check each item that is correct
   (II). Mark each incorrect or omitted answer with "x"
   (III). Give one point for each question
   (IV). When an exercise has more than one answer, give the proper fractional part of one for each correct answer (amount depending upon the number of parts)
   (V). Place the student's total score in the proper place, on the front page.

B. The Final Test

The preliminary test was scored and the results were tabulated. The tabulation helped to point out those questions that were misleading, too easy, or too difficult. The questions that
were either misleading, too easy, or too difficult were eliminated and other items were substituted for them. Very few changes were made. Therefore, the final forms were very similar to the preliminary forms.

1. Changes made in Form A
   a. Directions were given in exercise 4 to "answer in lowest terms."
   b. The order of exercises eleven and twelve were reversed because exercise eleven was more difficult than exercise twelve.
   c. The directions for exercises 14 to 19, inclusive, were reworded.
   d. Exercise 22 was reworded.
   e. The directions for exercises 24 to 27, inclusive, were omitted because they were not necessary.
   f. Another problem was substituted for exercise 24 because it was too easy.
   g. The direction for exercise 34 was reworded.
   h. A sign and a letter in exercise 38 were changed. The exercise as first given was too difficult.

2. Changes made in Form B
   a. Exercise 14 was changed from \( \frac{3.2a}{2.4} \) to \( \frac{3.2a}{0.8} \). Too many students read the decimal points as multiplication signs.
   b. \( \frac{x^2 - 2x - 3}{5} \) was substituted for \( 48a^2 - 6ab + \frac{1}{5}b^2 \) in
exercise 33. The expression, before revision, was puzzling to the students.

c. The negative sign was moved closer to the 6 in exercise 34.

d. One sign was changed in each exercise of 38 and 39.

The directions for scoring and evaluating the final forms were made by the Mathematics Contest Committee.6

6See Appendix A
CHAPTER V

I. AN ACHIEVEMENT TEST IN FIRST YEAR ALGEBRA

A. State High School Mathematics Contest, 1934

Identification Blank (Sectionals)

Identification Number (to be filled in by the director.)
Name of Contestant
School ______________________ Town ______________________
Name of your teacher ______________________
Underline the word which indicates your high-school standing:
  freshman, sophomore, junior, senior
Sectional at which you are competing ______________________

B. Directions to students

There are 63 questions in this test. Each question is weighted according to its difficulty. The time is limited to ninety minutes. Do not spend too much time on any one item, but work as accurately and as rapidly as you can.

1Identification blank as formulated by the Indiana State High School Mathematics Contest Committee.
INDIANA STATE—HORN—FIRST YEAR ALGEBRA TEST
1934

Form A Identification No.____ Score____

Directions for Exercises 1–6: Solve the equations and place your answers in the spaces at the right.

For example: \(3x + 2 = 8\) ........................................... \(x = \_2\)
1. \(15 - 2y - 3 = -5y\) ........................................... \(y = \_
2. \(2x = 9 - 3(x - 2)\) ........................................... \(x = \_
3. \(6.3x - 3.1x = 9.8 - 3.8x\) ........................................... \(x = \_
4. \(\frac{7}{5}x - \frac{1}{5}x = 4 - \frac{1}{5}x\) (Express answer in lowest terms) ........................................... \(x = \_
5. \(10 - (4x + 3)(8x + 3) = 3x - (16x + 25)(2x - 3)\) ........................................... \(x = \_

Do what each exercise says to do and place your results on the spaces at the right.

6. Add: \(x^2 - x + 3\), \(3x^2 + 6x - 5\), \(x^2 + x - 4\) ........................................... 6
7. Take \(5 - 6x + 8y\) from \(2 + 8x - 13y\) ........................................... 7
8. Find the product: \((9ab - 4)(2ab + 5)\) ........................................... 8
9. Simplify completely: \(6(3x - 4)^2 - 2(x + 1)^2\) ........................................... 9
10. Divide: \(28x^10 - 40x^7 + 24x^3 - 4x\) by \(4x\) ........................................... 10
11. Divide: \(4x^2 - 8ax + 3a^2\) by \(2x - 3a\) ........................................... 11
12. Divide: \(-8 + x^3 + x + 4x^2\) by \(x - 2\) ........................................... 12
13. Multiply: \((a + x + y)(a - x - y)\) ........................................... 13

Simplify the following 6 expressions by performing indicated operations and reducing fractions to lowest terms.

14. \(-\frac{8a^3b}{50ab^3}\) ........................................... 14
15. \(\frac{1}{2}x^3 + \frac{3}{4}x^2 + \frac{1}{2}x + \frac{1}{8}x^3 + \frac{1}{2}x^2 + x\) ........................................... 15
16. \(\frac{x^2 + 2x + 1}{x^2 - 1} \cdot \frac{5x}{x + 1}\) ........................................... 16
17. \(\frac{x^3 - 3x^2 + 2x}{3x^2 - 10x + 12}\) ........................................... 17
18. \(3x(2x^2 - 5x + 4) - 4x(x^2 + 3x - 6)\) ........................................... 18
19. \[ \frac{x^2 - 4y^2}{x + 3y} \div (xy - 2y^2) \]

Place the answer to each exercise on the space at the right.

20. Change this verbal expression into an algebraic equation: Five times a certain number diminished by 19 is equal to 2 times the number diminished by 7.

21. When the distance and the time are known write the formula for finding the rate, (use \( r \), \( d \), and \( t \)).

22. When the radius of a circle is doubled, the area of the larger circle is how many times as large as that of the smaller circle?

23. If the area and the width of a rectangle is known, write the formula for finding the length of the rectangle (use \( A \), \( l \), and \( w \)).

24. When \( S = 784 \), and \( g = 32 \), find the value of \( t \) from the formula \( S = \frac{1}{2}gt^2 \).

25. When \( P = \$522.00 \), \( r = \frac{41\%}{2} \), and \( t = 5 \) yrs., Find the value of \( A \) from the formula \( A = P + Prt \).

26. When \( I = \$55 \), \( P = \$400 \), and \( t = 2\frac{1}{2} \) yrs., Find the value of \( r \) from the formula \( I = Prt \).

27. When \( A = 107.2 \), \( a = 7.2 \), and \( b = 19.6 \), find the value of \( h \) from the formula \( A = \frac{h}{2}(a + b) \).

Find the prime factors of the following 6 expressions and place your results in the spaces to the right.

28. \( 9a^2 - 100 \)

29. \( x^2y^2 - 10xy + 25 \)

30. \( 21x^2 + 2x - 8 \)

31. \( 16x^2y - 35yz^2 \)

32. \( c^4 - d^4 \)

33. \( 50ax^2 - 140ax + 98a \)

Miscellaneous Exercises — Follow directions as given for each exercise and place the result in the space to the right.

34. Find the roots of the equation: \( \frac{x^2}{3} + \frac{x}{2} = \frac{9}{2} \)

35. Divide: \( \frac{-20x + 10x^2 - 5x^3}{-5x} \)
36. Solve for \( x \): \[
\frac{3x + 2}{11} = \frac{1}{4}(x + 2)
\]

\[ x = -\frac{8}{7} \]

37. Simplify: \[
\frac{4x^2 - 25y^2}{16x^2 - 9y^2} \cdot \frac{4x^2 + 3xy}{2xy - 5y^2}
\]

38. Add: \[
\frac{x - y}{x^2 + xy} + \frac{-2}{x}
\]

39. Combine into 1 fraction: \[
\frac{a + 2}{a^2 + 3a - 28} - \frac{5}{a - 4}
\]

Directions for exercises 40–49: Draw a line under the answer and put the number of that answer in the space at the right.

For example: A statement of the equality of two quantities is called (1) a formula (2) an equation (3) a polynomial (4) a binomial

40. Two angles are called complementary if their sum is (1) 90° (2) 180° (3) 45° (4) 120°

41. In the expression 2x, the exponent of x is (1) 0 (2) 1 (3) 2 (4) \( \frac{1}{2} \)

42. In the expression \( x^2 + 7 \) the coefficient of \( x^2 \) is (1) 7 (2) 0 (3) 2 (4) 1

43. The expression \( ax^2 + bx + c \) is a (1) monomial (2) linear equation (3) binomial (4) trinomial.

44. A number placed to the right and slightly above another to indicate a power is called (1) the monomial factor (2) the coefficient (3) an exponent (4) the product of the numbers.

45. The expression \( 5t + 2u \) is made up of (1) one term (2) 2 terms (3) 3 terms (4) 4 terms

46. In the formula \( P = 2(1 + w) \), the 2 is (1) a root (2) a variable (3) an exponent (4) a constant.

47. The point on a graph where the axes cross is called the (1) ordinate (2) abscissa (3) origin (4) coordinate.

48. The y-value of a given point on the graph is called the (1) abscissa (2) origin (3) ordinate (4) axis.

49. The dividend = divisor \cdot quotient + the (1) sum (2) Multiplicand (3) product (4) remainder.
Solve for each of the unknowns in the following sets of equations and place your answers in the spaces to the right.

50. \(6y + x = 3\) ........................................ \(x = \) 
\(8y + 2x = 3\) ........................................ \(y = \)

51. \(x + y = 3000\) ........................................ \(x = \) 
\(.06x + .03y = .04(x + y)\) ................................ \(y = \)

52. \(\frac{1}{x} + \frac{3}{y} = 11\) ................................ \(x = \) 
\(\frac{5}{x} + \frac{4}{y} = 23\) ................................ \(y = \)

Follow the directions of each problem and place your results in the proper places at the right.

53. If 5 times A's salary exceeds twice B's by $6400 and 4 times A's salary exceeds 3 times B's by $2600, find the salary of each man. 

\[\begin{align*}
\text{A's} &= \\
\text{B's} &=
\end{align*}\]

54. A farmer wishes to fence a rectangular field which is 3 times as long as it is wide. He wishes to use 160 rod of fencing. Find the length and width of the field. 

\[\begin{align*}
\text{Length} &= \\
\text{Width} &=
\end{align*}\]

55. A is 2 yrs. younger than B; but 6 yrs. ago 7 times B's age added to 5 times A's age was 110 yrs. Find their present ages. Write the equation...

\[\begin{align*}
\text{A's age} &= \\
\text{B's age} &=
\end{align*}\]

56. One pipe can fill a tank in 9 hrs. another in 5 hrs. If both pipes are used, how many hours are needed to fill the tank? Write the equation.

\[\text{Eq. =} \]

(Find results to the nearest tenth) Hrs. needed

57. After paying an agent 3% for selling his house, a man received $6111. What was the selling price of the house?

\[\text{Food} \quad \text{Clothes} \quad \text{Rent} \quad \text{Total} \]

58. A family spends twice as much for food as for rent, and $200 a year less for clothing than for rent. If these expenses total $2600, how much is spent for each item?

Food
Clothes
Rent

59. If 15 men are needed to complete a job in 10 days, how many men are needed to do the work in 3 days? Write the proportion.

Proportion

Men needed

60. If a bu. of oats is worth 33 cents and a bu. of corn is worth 45 cents, how many bu. of each should a miller use to make a mixture of 100 bu. worth 40¢ a bu? Write the equation.

\[\begin{align*}
\text{Eq. =} \quad \text{Oats} \quad \text{Corn}
\end{align*}\]
61. A boy has 6 nickels, 5 more dimes than nickels, and 3 times as many quarters as nickels. If the value of all the coins is $6.70, how many nickels, dimes, and quarters did he have? Write the equation.

\[ \begin{align*}
\text{Nickels} & \quad \text{Dimes} \quad \text{Quarters} \\
\text{Eq.} & \quad \text{Eq.} \\
\end{align*} \]

62. A part of $5000 is invested at 6% and the remainder at 6.5%. If the annual income from the second investment is $57.50 more than from the first, find how much is invested at each rate.

\begin{align*}
\text{Amt. at 6%} & \quad \text{Amt. at 6.5%} \\
\end{align*}

63. Graph this set of equations on the graph paper below using the same axes. Finish filling out the two tables of values. State the coordinates of the point of intersection.

\[ \begin{align*}
(1) \ y & = x + 1 \\
(2) \ y & = 2x - 2 \\
\end{align*} \]

\[
\begin{array}{c|c|c}
 x & y \\
-2 & -1 \\
-1 & 0 \\
0 & 1 \\
1 & 2 \\
2 & 3 \\
3 & 4 \\
\end{array}
\]

\[
\begin{array}{c|c|c}
 x & y \\
-2 & -1 \\
-1 & 0 \\
0 & 1 \\
1 & 2 \\
2 & 3 \\
3 & 4 \\
\end{array}
\]
Directions for exercises 1–5: Solve the equations and place your answers in the spaces at the right.

For example: \(5x + 1 = -9\) \(x = -2\)

1. \(5 - 3x = 7x + 13\) \(x = 1\)
2. \(-3x = 8 - (5 + 2x)\) \(x = 2\)
3. \(8(x + .25) - (3x + .75) = x + 6.25 - 6(2x - .5)\) \(x = 3\)
4. \(\frac{1}{2}(6x - 4) = \frac{1}{3}(6x - 9)\) \(x = 4\)
5. \(\frac{3x + 20}{6} + \frac{5}{3} = \frac{x + 16}{5} + \frac{2(6 + x)}{3}\) \(x = 5\)

Do what each exercise says to do and place your results in the spaces at the right.

6. Add: \(-3x + 2y - z\), \(5x - 3y + z\), \(+2x + 3y\) \(x = 6\)
7. Take: \(2a - 5b + 4c\) from \(-5a + 7b - 3c\) \(x = 7\)
8. Divide: \(20x^5 - 24x^4 + 12x^3 - 4x^2\) by \(4x^2\) \(x = 8\)
9. Simplify completely: \(3x(2x^2 - 5x + 4) - 4x(x^2 + 3x - 6)\) \(x = 9\)
10. Multiply: \((7a - 3b)^2\) \(x = 10\)
11. Multiply: \((2a - 3)(3a^2 - 7 + 4a)\) \(x = 11\)
12. \((12a^3 - 11a^2b + 8ab^2 - 4b^3)\) \(÷ (3a - 2b)\) \(x = 12\)
13. Divide: \(8x^3 + y^3\) by \(2x + y\) \(x = 13\)

Simplify completely the following 6 expressions. Place your results in the spaces at the right.

14. \(\frac{3.2a}{.8}\) \(x = 14\)
15. \(-4(a + 1) - 3(5 - 3a)\) \(x = 15\)
16. \(\frac{a^2 - ab}{a^2 - b^2}\) \(x = 16\)
17. \(18\left(\frac{1}{3}a - \frac{1}{6}b + \frac{1}{2}c\right) - 12\left(\frac{1}{4}a + \frac{1}{2}b - \frac{1}{6}c\right)\) \(x = 17\)
18. \(\frac{x^2 + 2xy + y^2}{x^4 - y^4} ÷ \frac{x^2y + xy^2}{x^2y + y^3}\) \(x = 18\)
19. \(\frac{a^2 + 4a - 12}{a^2 - a - 12} \div \frac{a^2 - 4a}{a^2 + 6a^2}\) \(x = 19\)
Place the result of each exercise in the space at the right.

20. Change this verbal expression into an algebraic equation. Ten added to 4 times a number is equal to 9 times that number diminished by 15. ........................................ 20

21. Write the formula for the number of tons, T, of coal that a bin will hold if 1 ton of coal occupies 38 cu. ft. ................................................................. T= ______________ 21

22. When the cost and the amount of profit on an article, is known, write the formula for finding the selling price. ................................................................. S.P.= ______________ 22

23. Write the formula for finding the principal, when the rate, time, and interest is given. (use i,P,r,t,) P= ______________ 23

Evaluation: Do as each exercise directs you to do and place your results in the spaces at the right.

24. When C = + 40, find the value of F from the formula F = 9/5C + 32 ........................................ F= ______________ 24

25. When P = 34, and 1 = 7, find the value of w from the formula P = 2(1 + w) ........................................ w= ______________ 25

26. When A = 107.2 in., b = 19.6 in. and a = 7.3 in., find the value of h from the formula A = h/2(a + b) ........................................ h= ______________ 26

27. When A = $302.50, P = $250 and t = 3 1/2 yrs., find the value of r from the formula A = P + Prt. ........................................ r= ______________ 27

Find the prime factors of the following 6 exercises and place your results in the spaces at the right.

28. \( \frac{x^2}{16} - \frac{1}{4} \) ........................................ ______________ 28

29. \( r^3 - 12r^2 + 32r \) ........................................ ______________ 29

30. \( 28y^3 - 22y^2 + 4y \) ........................................ ______________ 30

31. \( a^4 - 16 \) ........................................ ______________ 31

32. \( x^4 - 8x^2 + 16 \) ........................................ ______________ 32

33. \( \frac{x^2}{5} - \frac{2x}{5} - \frac{3}{5} \) ........................................ ______________ 33

Miscellaneous Exercises: Follow directions as given for each exercise and place the results in the spaces at the right.

34. Simplify completely: \( -6ab \) \( \frac{15a^3b}{18ab^2} \) ........................................ ______________ 34
35. Add: \[ \frac{x-5}{x^2-1} + \frac{2x-7}{6x-6} \] .......................... 35

36. Solve for \( x \): \[ \frac{3x}{7} = 30 - \frac{2(58-x)}{5} \] .......................... 36

37. Find the roots of the equation: \( 3x^2 - 8x + 4 = 0 \) .......................... 37

38. Divide: \( (3 - \frac{2}{x+1}) \div (9 + \frac{8}{x^2-1}) \) .......................... 38

39. Solve for \( x \): \[ \frac{5x}{x+3} - \frac{13}{2x+6} = \frac{5x-7}{3x+9} \] \( \ldots \ldots \) \( x= \) .......................... 39

Directions for exercises 40-49: Draw a line under the right answer and put the number of that answer in the proper space at the right.

For example: The graph of a linear equation is always (1) a straight line (2) a broken line (3) a curve line (4) two intersecting lines. \( \ldots \) .......................... 1

40. The expression \( ab^2c \) is (1) a trinomial (2) a polynomial (3) a monomial (4) an identity. .......................... 40

41. An algebraic expression in which the parts are separated by the signs + or - is called (1) a monomial (2) a prime number (3) an identity (4) a polynomial .......................... 41

42. Two angles are called supplementary if their sum is (1) 90° (2) 60° (3) 180° (4) 120° .......................... 42

43. In the expression \( 4x \) the \( x \) is (1) a constant (2) a root (3) an exponent (4) a variable .......................... 43

44. In the expression \( 5a^3 \) the figure 3 is the (1) base (2) exponent (3) coefficient (4) root .......................... 44

45. In the expression \( 2x^3 \), the 2 is the (1) coefficient (2) base (3) exponent (4) root .......................... 45

46. The \( x \)-value of a given point on the graph is called the (1) ordinate (2) coordinate (3) abscissa (4) origin .......................... 46

47. If the product of two or more factors is zero then at least one of the factors must be (1) negative (2) zero (3) a fraction (4) positive .......................... 47

48. The expression \( 6h^2 \) means (1) \( 6h \cdot 6h \) (2) \( 6 \cdot 6h \) (3) \( 6h \cdot h \) (4) \( (6h)^2 \) .......................... 48

49. The two numbers which locate a point on the graph are called (1) axes (2) abscissa (3) origin (4) coordinates .......................... 49
Solve for each of the unknowns in the following sets of equations and place your answers in the spaces at the right.

50. \[6x + 5y = -3\] \[9y - 2x = 1\] \[x=\] \[y=\]

51. \[5 - 2(r + s) = 3\] \[r + 3(r - s) = 18\] \[r=\] \[s=\]

52. \[a - 10 = \frac{5}{3}(b - 10)\] \[a + 20 = \frac{4}{3}(b + 20)\] \[a=\] \[b=\]

Follow the directions as given for each problem and place your results in the proper spaces at the right.

53. A dealer wishes to sell hats at $4.50. At what price must he buy them to make 25% on the cost? Price

54. A man wishes to fence a rectangular field. The ratio of the dimensions is to be 3 to 2. He wishes to use 60 rod of fencing. What are the dimensions of the field? \[L=\] \[W=\]

55. A post 33 feet long is to be cut into two pieces so that one piece will be 3/5 of the other piece. How long is each piece? \[\text{Short piece}\] \[\text{Long piece}\]

56. A's salary exceeds twice B's salary by $500. And 6 times A's salary exceeds 8 times B's by $7200. Find the salary of each man. Write the equation or equations.

57. The sum of the circumference and the diameter of a circle is 116 in. Find the radius. (Use 22/7 for \(\pi\)) Write the equation or equations.

58. A post is 4 ft. 6 in. tall and casts a shadow 2 feet long. At the same time the shadow of a building is 352 ft. long. How tall is the building? Write the proportion.

59. The cost of one kind of coffee is 6 cents more a pound than the cost of another kind. If 50 lb. of the better kind and 70 lb. of the cheaper kind will make a mixture worth 25¢ a pound, how much per pound does each kind cost? Write the equation or equations.\[\text{Eq.}\]

60. John and Frank, together, can do a piece of work in 30 hrs. John needs 32 hr. more than Frank does to do the work. How long does it take each one to do the work alone? John \[\text{Frank}\]
61. A farm hand was to be paid $2 a day for the days he worked. He was to forfeit 70¢ a day for his board for each day that he was idle. After 60 days he received $106.50. How many days did he work and how many days was he idle? Days worked Days idle

62. The cost of a trip for a number of men was $180, and was to be shared equally. Because 5 men failed to appear, however, the cost per person was increased $2.00. What was the original number of men? Men

63. Graph this set of equations on the graph paper below using the same axes. Finish filling out the two tables of values. State the coordinates of the point of intersection.

\[ \begin{align*}
(1) & \quad y - 3x = 0 \\
(2) & \quad y + 2x + 4 = 0 \\
\end{align*} \]
CHAPTER VI

I. VALIDITY AND RELIABILITY OF THE TEST

A. Validity

The most important single fact which can be known about a test is the degree of validity which it possesses. Validity may be defined variously; these separate definitions constituting collectively, the ideas incorporated in the term.

1. Definitions of validity
   a. Validity is the degree to which a test or examination measures what it is intended to measure.
   b. Validity is the general worthwhileness of an examination.
   c. Validity refers to the care taken to incorporate in a test those elements or items which are of prime importance, and to the pains taken to eliminate the non-essential.
   d. Validity is in general the degree to which a test parallels the curriculum and good teaching practice.
   e. Validity refers to the value of the test for measuring specific abilities in an accurate fashion, and a test ceases to have validity when applied to the measurement of abilities for which it was not intended.

Validity includes reliability. That is, a valid test must
of necessity be a reliable test." 1

2. Methods used to validate the test

a. One of the sectional group of papers was chosen as a basis for determining the validity of the test. This sectional group of papers was chosen because there was a gradual decline in the scores from the highest to the lowest score. The median score was found and the papers were divided into two groups; those papers above the median were called the "good" group, those papers on or below the median were called the "poor" group. The scores were taken for each item in each group and totaled. The scores were expressed in percent.

"The best items will show the largest difference in successes in favor of the "good" group." 2 The "good" group made a larger score than the "poor" group on fifty-one items (81.95 per cent), an equal score on eight of the items (12.7 per cent), and the "poor" group made a larger score than the "good" group on four of the items (5.35 per cent).

A study of the tabulation seemed to show that four of the items in the test were too easy and five

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1G. M. Ruch, *The Objective or New-Type Examination*. (New York: Scott, Foresman and Co. 1929), pp. 27, 28.

2Ibid., p. 37.
of the items were too difficult to measure what they were supposed to measure. Items, 7, 9, 14, and 48 were too easy. Items, 24, 54, 55, 60, and 62 were too difficult.

The items that are too easy or too difficult are doubtful as valid items. However, a study of these items showed that three of the four items which are too easy come at the first or near the first items in their group. The fourth item is located in the multiple-choice section. Four of the five items which are too difficult come in the section of written problems. The other item seems to be misplaced as it is located at the beginning of its group when it should be nearer the end.

The test should not be criticized too severely because of these nine items. Some easy and some difficult items are necessary in order to measure the weaker and better students. The writer had no way determining the ability of the students beforehand. Some easy and some difficult items are necessary to test the various abilities. These items probably would not have been too easy or too difficult if an average group was tested.

Summary: In comparing the per cents of the "good" and the "poor" group the writer found that 81.95 per cent of the items showed successes in favor of the "good" group, 12.7 per cent of the
items showed no difference between the two groups, and 5.35 per cent of the items showed successes in favor of the "poor" group.

According to the too-easy and too-difficult items, four items were too easy, and five items were too difficult, making ninety per cent of the items valid.

When consideration is given to the necessity and proper allocation of easy and difficult items, all but one of the nine items may be considered valid, making sixty-two out of the sixty-three items, or 98.4 per cent of the items, valid.

The above data would indicate that the test is valid.

b. Tables III and IV show the proposed and actual distribution of questions by units for the first-year algebra test, Form A and Form B respectively. Such tables aid in making the test more valid.

c. The test items of this study were checked and approved by two of the committee. The test, Form A and Form B, was accepted and given by the Mathematics Contest Committee or their representatives.
### TABLE III

**TABLE OF SPECIFICATIONS SHOWING THE PROPOSED AND ACTUAL DISTRIBUTION OF QUESTIONS BY UNITS FOR THE FIRST YEAR ALGEBRA TEST, FORM A**

<table>
<thead>
<tr>
<th>Units</th>
<th>Number of Pages for Each Unit</th>
<th>Per Cent of Pages</th>
<th>Proposed Number of Questions</th>
<th>Actual Number of Questions</th>
</tr>
</thead>
<tbody>
<tr>
<td>Literal numbers</td>
<td>22</td>
<td>9.3</td>
<td>6</td>
<td>7</td>
</tr>
<tr>
<td>Positive and negative numbers</td>
<td>25</td>
<td>10.6</td>
<td>7</td>
<td>3</td>
</tr>
<tr>
<td>Linear equations</td>
<td>21</td>
<td>8.9</td>
<td>5</td>
<td>10</td>
</tr>
<tr>
<td>The solution of problems</td>
<td>30</td>
<td>12.7</td>
<td>8</td>
<td>6</td>
</tr>
<tr>
<td>Equations containing fractions</td>
<td>21</td>
<td>8.9</td>
<td>5</td>
<td>5</td>
</tr>
<tr>
<td>Ratio and proportion</td>
<td>12</td>
<td>5.1</td>
<td>3</td>
<td>3</td>
</tr>
<tr>
<td>Graphs</td>
<td>14</td>
<td>5.9</td>
<td>4</td>
<td>3</td>
</tr>
<tr>
<td>Sets of linear equations</td>
<td>18</td>
<td>7.6</td>
<td>5</td>
<td>6</td>
</tr>
<tr>
<td>Formulas</td>
<td>12</td>
<td>5.1</td>
<td>3</td>
<td>2</td>
</tr>
<tr>
<td>Multiplication and factoring</td>
<td>33</td>
<td>14.0</td>
<td>9</td>
<td>9</td>
</tr>
<tr>
<td>Division and fractions</td>
<td>28</td>
<td>11.9</td>
<td>8</td>
<td>9</td>
</tr>
<tr>
<td><strong>Total</strong></td>
<td><strong>236</strong></td>
<td><strong>100.0</strong></td>
<td><strong>63</strong></td>
<td><strong>63</strong></td>
</tr>
</tbody>
</table>
TABLE IV

TABLE OF SPECIFICATIONS SHOWING THE PROPOSED AND ACTUAL DISTRIBUTION OF QUESTIONS BY UNITS FOR THE FIRST YEAR ALGEBRA TEST, FORM B

<table>
<thead>
<tr>
<th>Units</th>
<th>Number of Pages for Each Unit</th>
<th>Per Cent of Pages</th>
<th>Proposed Number of Questions</th>
<th>Actual Number of Questions</th>
</tr>
</thead>
<tbody>
<tr>
<td>Literal numbers</td>
<td>22</td>
<td>8.9</td>
<td>6</td>
<td>6</td>
</tr>
<tr>
<td>Positive and negative numbers</td>
<td>25</td>
<td>10.2</td>
<td>6</td>
<td>3</td>
</tr>
<tr>
<td>Linear equations</td>
<td>21</td>
<td>8.5</td>
<td>5</td>
<td>10</td>
</tr>
<tr>
<td>The solution of problems</td>
<td>30</td>
<td>12.2</td>
<td>8</td>
<td>4</td>
</tr>
<tr>
<td>Equations containing fractions</td>
<td>21</td>
<td>8.5</td>
<td>5</td>
<td>6</td>
</tr>
<tr>
<td>Ratio and proportion</td>
<td>12</td>
<td>4.9</td>
<td>3</td>
<td>3</td>
</tr>
<tr>
<td>Graphs</td>
<td>14</td>
<td>5.7</td>
<td>4</td>
<td>3</td>
</tr>
<tr>
<td>Sets of linear equations</td>
<td>18</td>
<td>7.3</td>
<td>5</td>
<td>6</td>
</tr>
<tr>
<td>Formulas</td>
<td>12</td>
<td>4.9</td>
<td>3</td>
<td>2</td>
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<tr>
<td>Multiplication and factoring</td>
<td>33</td>
<td>13.4</td>
<td>8</td>
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<tr>
<td>Division and fractions</td>
<td>30</td>
<td>12.2</td>
<td>8</td>
<td>7</td>
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<tr>
<td>Fractional equations</td>
<td>8</td>
<td>3.3</td>
<td>2</td>
<td>4</td>
</tr>
<tr>
<td><strong>Total</strong></td>
<td><strong>246</strong></td>
<td><strong>100.0</strong></td>
<td><strong>63</strong></td>
<td><strong>63</strong></td>
</tr>
</tbody>
</table>
TABLE V

DISTRIBUTION OF THE STATE MATHEMATICS SCORES IN FIRST YEAR ALGEBRA, FORM B

<table>
<thead>
<tr>
<th>State Rank</th>
<th>Scores State</th>
<th>Scores Sectional</th>
<th>Students' Initials</th>
<th>Identification Number</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>305</td>
<td>274</td>
<td>R. T.</td>
<td>44</td>
</tr>
<tr>
<td>2</td>
<td>297</td>
<td>281</td>
<td>M. H.</td>
<td>45</td>
</tr>
<tr>
<td>3</td>
<td>290</td>
<td>277</td>
<td>B. W.</td>
<td>21</td>
</tr>
<tr>
<td>4</td>
<td>285</td>
<td>293</td>
<td>H. O. D.</td>
<td>26</td>
</tr>
<tr>
<td>5</td>
<td>285</td>
<td>307</td>
<td>R. M.</td>
<td>36</td>
</tr>
<tr>
<td>6</td>
<td>263</td>
<td>287</td>
<td>R. V. F.</td>
<td>38</td>
</tr>
<tr>
<td>7</td>
<td>278</td>
<td>302</td>
<td>M. J. C.</td>
<td>51</td>
</tr>
<tr>
<td>8</td>
<td>278</td>
<td>276</td>
<td>J. L. M.</td>
<td>37</td>
</tr>
<tr>
<td>9</td>
<td>274</td>
<td>261</td>
<td>M. D.</td>
<td>6</td>
</tr>
<tr>
<td>10</td>
<td>274</td>
<td>260</td>
<td>E. G.</td>
<td>11</td>
</tr>
<tr>
<td>11</td>
<td>272</td>
<td>286</td>
<td>J. W.</td>
<td>27</td>
</tr>
<tr>
<td>12</td>
<td>270</td>
<td>271</td>
<td>J. G.</td>
<td>8</td>
</tr>
<tr>
<td>13</td>
<td>269</td>
<td>272</td>
<td>M. P. M.</td>
<td>14</td>
</tr>
<tr>
<td>14</td>
<td>267</td>
<td>275</td>
<td>R. B.</td>
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</tr>
<tr>
<td>15</td>
<td>267</td>
<td>275</td>
<td>H. K.</td>
<td>46</td>
</tr>
<tr>
<td>16</td>
<td>267</td>
<td>275</td>
<td>H. C.</td>
<td>5</td>
</tr>
<tr>
<td>17</td>
<td>261</td>
<td>302</td>
<td>H. J. W.</td>
<td>50</td>
</tr>
<tr>
<td>18</td>
<td>261</td>
<td>289</td>
<td>H. W.</td>
<td>3</td>
</tr>
<tr>
<td>19</td>
<td>260</td>
<td>280</td>
<td>M. H.</td>
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</tr>
<tr>
<td>20</td>
<td>260</td>
<td>290</td>
<td>B. P.</td>
<td>41</td>
</tr>
<tr>
<td>21</td>
<td>258</td>
<td>281</td>
<td>M. M. M.</td>
<td>52</td>
</tr>
<tr>
<td>22</td>
<td>257</td>
<td>275</td>
<td>C. H.</td>
<td>24</td>
</tr>
<tr>
<td>23</td>
<td>253 (\frac{1}{2})</td>
<td>257</td>
<td>G. A. K.</td>
<td>55</td>
</tr>
<tr>
<td>24</td>
<td>253 (\frac{3}{4})</td>
<td>280</td>
<td>H. R. R.</td>
<td>25</td>
</tr>
<tr>
<td>25</td>
<td>252</td>
<td>239</td>
<td>D. F.</td>
<td>33</td>
</tr>
<tr>
<td>26</td>
<td>251</td>
<td>276</td>
<td>J. N. S.</td>
<td>43</td>
</tr>
<tr>
<td>27</td>
<td>251</td>
<td>269</td>
<td>H. Z.</td>
<td>31</td>
</tr>
<tr>
<td>28</td>
<td>250</td>
<td>262</td>
<td>M. D.</td>
<td>39</td>
</tr>
<tr>
<td>29</td>
<td>249</td>
<td>277</td>
<td>D. S.</td>
<td>47</td>
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<tr>
<td>30</td>
<td>247</td>
<td>278</td>
<td>J. C.</td>
<td>29</td>
</tr>
<tr>
<td>31</td>
<td>242</td>
<td>283</td>
<td>E. H.</td>
<td>10</td>
</tr>
<tr>
<td>32</td>
<td>242</td>
<td>275</td>
<td>J. R.</td>
<td>7</td>
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<tr>
<td>33</td>
<td>240</td>
<td>265</td>
<td>M. A. W.</td>
<td>54</td>
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TABLE V (continued)

<p>| | | | | |</p>
<table>
<thead>
<tr>
<th></th>
<th></th>
<th></th>
<th></th>
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</thead>
<tbody>
<tr>
<td>26</td>
<td>239</td>
<td>303</td>
<td>J. P.</td>
<td>15</td>
</tr>
<tr>
<td>27</td>
<td>232</td>
<td>298</td>
<td>L. M.</td>
<td>35</td>
</tr>
<tr>
<td>28</td>
<td>231</td>
<td>268</td>
<td>E. J.</td>
<td>23</td>
</tr>
<tr>
<td>29</td>
<td>230</td>
<td>299</td>
<td>J. L. P.</td>
<td>4</td>
</tr>
<tr>
<td>30</td>
<td>229</td>
<td>277</td>
<td>L. H.</td>
<td>28</td>
</tr>
<tr>
<td>30</td>
<td>229</td>
<td>285</td>
<td>L. P.</td>
<td>34</td>
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<tr>
<td>31</td>
<td>228</td>
<td>289</td>
<td>R. S.</td>
<td>13</td>
</tr>
<tr>
<td>31</td>
<td>228</td>
<td>271</td>
<td>M. W.</td>
<td>42</td>
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<tr>
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<td>227</td>
<td>273</td>
<td>E. A.</td>
<td>48</td>
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<td>269</td>
<td>R. S.</td>
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<tr>
<td>33</td>
<td>225</td>
<td>261</td>
<td>M. E.</td>
<td>12</td>
</tr>
<tr>
<td>33</td>
<td>225</td>
<td>264</td>
<td>D. Q. R.</td>
<td>17</td>
</tr>
<tr>
<td>34</td>
<td>224</td>
<td>261</td>
<td>E. K.</td>
<td>19</td>
</tr>
<tr>
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<td>222</td>
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<td>J. W. G.</td>
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</tr>
<tr>
<td>36</td>
<td>221</td>
<td>280</td>
<td>I. S.</td>
<td>30</td>
</tr>
<tr>
<td>37</td>
<td>213</td>
<td>284</td>
<td>W. P.</td>
<td>49</td>
</tr>
<tr>
<td>38</td>
<td>214</td>
<td>271</td>
<td>P. M.</td>
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<td>39</td>
<td>203</td>
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<td>C. E. B.</td>
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<td>287</td>
<td>E. S.</td>
<td>32</td>
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<tr>
<td>41</td>
<td>194</td>
<td>283</td>
<td>C. H. C.</td>
<td>9</td>
</tr>
</tbody>
</table>

Report of the Mathematics Contest Committee on the State Contest Results for Algebra.

d. TABLE V gives the state rank, state and sectional scores, initials, and identification number of each student. State test scores ranged from 194 to 305. The highest possible score for the sectional test (Form A) was 339. The highest possible score for the state test (Form B) was 340. No student made the highest possible score on either test. Fifty-three students competed in the state contest.

TABLE V is another criterion of the validity of the test.
B. Reliability

"Reliability is second only to validity as a criterion of the worth of a test or examination. We might say that the second most important fact which we can know about a test is the degree of reliability which it possesses. A number of statements are given below, which, collectively and individually, serve to define the concept.

1. Definition of reliability
   a. Reliability refers to the degree to which a test measures what it does measure; not necessarily what it is claimed to measure.
   b. Reliability refers to the degree of accuracy of measurement.
   c. Reliability refers to the amount of confidence that may be placed in the mark or score on a test as a measure of some ability of a pupil.
   d. Reliability is one aspect of validity. A valid test is necessarily reliable, but a reliable test need not have high validity at all for a particular purpose.
   e. Reliability refers to the stability of an estimate of a pupil's ability from one sampling to another.

2. Methods of insuring reliability
   a. Objectivity of scoring, or evaluating.
   b. Character of the sampling included in the test items."³

³G. M. Ruch, op. cit., pp. 40-42.
3. Determining the reliability of the test

a. For determining the reliability of the test the chance-halves method was used. One hundred forty-seven cases were selected from seven sectionals. The sectionals were picked at random. The only practice followed was to use all the papers from the sectionals selected. From these papers the correct scores of the odd-numbered questions were compared with the correct scores of the even-numbered ones. The formula as given by Ruch\(^4\) was used to find \( r \). The results as given by this formula is to be interpreted as the reliability of either half of the examination. What is needed is the reliability of the whole examination. To obtain the reliability of both parts of the examination the Spearman-Brown formula\(^5\) was used. This gave a coefficient of reliability of .835 for Form A. The reliability coefficient of Form B was .582. The reliability coefficient for Form B was double checked by the statistics class.

The coefficient of reliability for Form A is considered fairly satisfactory. According to Ruch's Table 81,\(^6\) it would take twice as many items in

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order to bring the reliability coefficient up to .90. The group being a select group would tend to lower the reliability, as the textbook on which the test was based was made largely for the average student.

The figures indicate that Form B was not very reliable. According to Ruch's Table 81, it would take about 275 items to bring the reliability coefficient up to .90. The test would be too long for the purpose it was to serve.

The reason for this low coefficient of reliability is probable due to the fact that test (Form B) was taken by starred mathematics students all over the state and their variation is probable due to mere computational errors rather than to mathematical reasoning.

7G. M. Ruch, op. cit., p. 421.
CHAPTER VII
I. CONCLUSIONS

The building of an achievement test does not provide a very fertile field for drawing conclusions. There are few definite results except the one chief objective—the finished test. However, the writer wishes to mention the invaluable experience which he, himself, has received in constructing tests. Going through the various methods books, textbooks, courses of study and tests has given the author a better knowledge of algebra. Making the list of objectives in algebra gave the writer a better knowledge of what the aims of algebra should be.

The best suggestion this study has to offer on test construction is to cite those interested to the work of Ruch.¹

The test revealed certain defects in teaching. Exercise 15 in Form A was the outstanding example which showed defective teaching, or lack of thought on the part of the students. The following expression was to be simplified: \( \frac{1}{2}x^3 + \frac{3}{4}x^2 + \frac{1}{2}x + \frac{1}{8}x^3 + \frac{1}{2}x^2 + x \). A great number of students worked this exercise incorrectly. Most of those who missed it made the same mistake; namely, they tried to simplify it the same way they would simplify an equation. As a result, they got eight times as much as they should have got. The students changed the denominators to a common denominator, added the like terms, and discarded the common denominator altogether, getting eight times too much.

It is believed that this test has served its purpose;

namely, to take care of the sectional and final examinations of the Indiana State Mathematics Contest for the school year 1933–34.

The test may be of value to those algebra teachers who are using Nyberg's First Course in Algebra (Revised Edition) as a textbook.
Contests in first year algebra and in plane geometry will be conducted this year by the Mathematics section of the State Teachers' Association in cooperation with the Extension Division of Indiana University.

Contest Committee. The following committee was appointed by the Mathematics section to formulate rules for the contests:

Mr. A. M. Welchons, Technical High School, Indianapolis, chairman
Mr. E. D. Hilderbrand, Greenfield High School, Greenfield
Mr. Clarence Lane, Jefferson High School, Lafayette
Mr. Walter O. Shriner, State Teachers College, Terre Haute
Mr. Arthur Sims, Lebanon High School, Lebanon
Mr. H. N. Whittern, Central High School, Muncie
Mrs. Adela Bittner, Extension Division, Indiana University, Bloomington

Enrollment. Enrollment is open to any high school in Indiana which fills out an enrollment blank and returns it with the enrollment fee to Indiana University before March first, 1934. The enrollment fee is one dollar per school, whether the school enrolls for one contest or for both contests.

Dates. March 1, 1934, final date for enrolling
April 7, 1934, sectional contests
April 20, 1934, state finals at Bloomington

Local Selection and Number of Contestants. The representatives from the local high school will be selected by the local school authorities on the basis of grades or by local tests as the local authorities may decide. The number of contestants for either the algebra or the geometry contest will be determined as follows:

2 from each school with an enrollment of 500 or less, and one additional contestant for each additional 1,000 or major fraction thereof.

Contest Series. Sectional contests will be held at approximately twenty centers. The list of centers is included with the enrollment blanks so that the school may select the most convenient center.

The fifty contestants in each contest who make the highest grades will compete in the finals, except that the winner of first place at each sectional will be eligible even though he does not place in the first fifty.

Length of Examination. The algebra examination will be a ninety
minute examination. The directors of the sectionals will be instructed to schedule the examinations at 10:00 a.m. in order that time may be allowed in the morning for the contestants to reach the examination centers and in the afternoon for the marking of the papers.

Questions. The questions will be prepared by Indiana mathematics teachers selected by the Mathematics contest committee. Copy for the questions and keys will be sent by the question making committees to the Extension Division of Indiana University not later than March first. The Extension Division will distribute the questions to the directors in charge of the sectionals.

Grading and Filing of Manuscripts. The manuscripts will be marked by graders at the sectionals and then sent to the Extension Division of Indiana University. The Extension Division will report all grades; no grades will be announced by the sectional directors. The Extension Division will file the manuscripts and identification cards for reference.

Awards. A certificate of merit will be awarded to the winner in each sectional. A gold, a silver, and a bronze medal will be awarded by the Extension Division to the three in each contest who make the highest ratings in the finals.

Expenses. All expenses for transportation, entertainment, etc., must be borne by the contestants or by the schools they represent.

ALGEBRA ANNOUNCEMENTS
(Fourth Annual Contest)

Eligibility. All pupils selected to represent the schools must have completed or must be taking first year algebra during the current school year. Note that:

Juniors and Seniors are not eligible; pupils taking third semester algebra are not eligible.

Content and Scope. The algebra contest will be confined to first year algebra. The contest will conform to the principles outlined in the state course of study in mathematics issued by the State Department of Public Instruction. The test will be based on Nyberg's First Course in Algebra (Revised Edition).

Final test: Through page 252.

Questions. The algebra test will be formulated by a committee of algebra teachers. The question makers are instructed to make a test longer than any student can complete in the ninety minute period; to take into account in making the questions the best of the new-type tests; to include some true-false, matching and selection exercises (not more than 10 percent of the contest); to weight the problems according to difficulty and importance;
to avoid ambiguous problems; to adhere strictly to the limits for both sectionals and finals; to indicate on the examination the complete possible score and the score for each question; and to prepare an answer key carefully correlated with the questions.

**Equipment.** Each student must bring to the examination one or two well-sharpened pencils.

### Tentative Sectional Centers.

<table>
<thead>
<tr>
<th>Location</th>
<th>Institution</th>
</tr>
</thead>
<tbody>
<tr>
<td>Angola</td>
<td>Extension Division, Bloomington</td>
</tr>
<tr>
<td>Ball State, Muncie</td>
<td>Extension Division, Fort Wayne</td>
</tr>
<tr>
<td>Indianapolis</td>
<td>Logansport</td>
</tr>
<tr>
<td>Central Normal College, Danville</td>
<td>Manchester College, North Manchester</td>
</tr>
<tr>
<td>Wabash College, Crawfordsville</td>
<td>Marion College, Marion</td>
</tr>
<tr>
<td>Depauw University, Greencastle</td>
<td>Purdue University, Lafayette</td>
</tr>
<tr>
<td>Earlham College, Richmond</td>
<td>Rensselaer</td>
</tr>
<tr>
<td>Evansville College, Evansville</td>
<td>University of Notre Dame, S.Bend</td>
</tr>
<tr>
<td>Franklin College, Franklin</td>
<td>Valparaiso University, Valparaiso</td>
</tr>
<tr>
<td>Greensburg</td>
<td>Vincennes University, Vincennes</td>
</tr>
<tr>
<td>Hanover College, Hanover</td>
<td></td>
</tr>
<tr>
<td>Indiana State Teachers College, Terre Haute</td>
<td></td>
</tr>
</tbody>
</table>
B. BIBLIOGRAPHY

1. General


Ruch, G. M. *The Objective or New-Type Examination*. Chicago: Scott, Foresman and Co., 1929.


2. Sources

a. Courses of Study

(I). City


Lorain, Ohio. *Courses of Study for Junior High Schools.* 1923.


Trinidad, Colorado. *Tentative Course of Study Trinidad Public Schools.* Trinidad, Colorado. 1928.

(II). State


Oregon. Courses of Study for the High Schools of Oregon. Issued by the State Department of Public Education, 1929.


b. Methods Books


"The Reorganization of Mathematics in Secondary Education."


c. Textbooks


Engelhardt, Fred, and Haertter, Leonard D. *First Course in
C. DIRECTIONS FOR GRADING THE MANUSCRIPTS

State High School Mathematics Contest, 1934

Please make sure that all papers are graded by competent graders. The graders should include at least one mathematician, as a referee to whom questions about the grading may be referred. Use red or blue pencil. Use X to mark every incorrect or omitted answer or method, and C to mark every correct answer, placing the mark on the right hand side, and next to it in the right hand margin the score for each exercise in a circle, thus, \( \textcircled{5} \), using red or blue pencil.

If you have an adding machine, add the scores, and clip the summation to the questions. Do not attempt to sum up without an adding machine.

If it is not possible to use an adding machine at the sectional, the summing up will be done by the Extension Division after the papers are returned. Do not announce any scores.

When the papers are graded, wrap them securely together with the identification blanks, and return them to the Extension Division, by express collect. Please be sure to return also the unused copies of the questions and the keys.

If questions arise about the interpretation of the key, please make a note of such instances on a separate sheet of paper and mail it to me when you send the manuscripts.
## D. Answer Key for Form A

Indiana State—Horn—First Year Algebra Test
1934

Form A

<table>
<thead>
<tr>
<th>Page 1</th>
<th>Key</th>
<th>Total Possible Score: 339</th>
</tr>
</thead>
<tbody>
<tr>
<td>(1) ( y = -4 )</td>
<td>(2) ( 5n - 19 = 2n - 7 ) (some other letter may be used)</td>
<td>(3)</td>
</tr>
<tr>
<td>(2) ( x = 3 )</td>
<td>(3)</td>
<td>(3)</td>
</tr>
<tr>
<td>(3) ( x = 1.4 ) or ( \frac{12}{5} )</td>
<td>(4) ( r = \frac{d}{t} )</td>
<td>(3)</td>
</tr>
<tr>
<td>(4) ( x = 8 )</td>
<td>(3) ( 4 )</td>
<td>(5)</td>
</tr>
<tr>
<td>(5) ( x = -2 )</td>
<td>(6) ( t = 7 )</td>
<td>(6)</td>
</tr>
<tr>
<td>(6) ( 5x^2 + 6x - 6 )</td>
<td>(2) ( 24 )</td>
<td>(3)</td>
</tr>
<tr>
<td>(7) (- 3 + 14x - 21y )</td>
<td>(3) ( \frac{A}{W} )</td>
<td>(3)</td>
</tr>
<tr>
<td>(8) ( 18a^2b^2 + 37ab - 20 )</td>
<td>(2) ( r = \frac{5}{2} % )</td>
<td>(5)</td>
</tr>
<tr>
<td>(9) ( 2(26x^2 - 74x + 47) ) or ( 52x^2 - 148x + 94 )</td>
<td>(5) ( 27 ) ( h = 8 )</td>
<td>(5)</td>
</tr>
<tr>
<td>(10) ( 7x^9 - 10x^6 + 6x^2 - 1 )</td>
<td>(3) ( 28 ) ( 3a - 10)(3a + 10) )</td>
<td>(4)</td>
</tr>
<tr>
<td>(11) ( 2x - a )</td>
<td>(3) ( 29 ) ( (xy - 5)^2 ) or ( (xy - 5)(xy - 5) )</td>
<td>(4)</td>
</tr>
<tr>
<td>(12) ( x + 6x + 13 + \frac{16}{x-2} )</td>
<td>(5) ( 30 ) ( (3x + 2)(7x - 4) )</td>
<td>(4)</td>
</tr>
<tr>
<td>(13) ( a^2 - x^2 - 2xy - y^2 )</td>
<td>(4) ( 31 ) ( y(4x + 5z)(4x - 5z) )</td>
<td>(6)</td>
</tr>
<tr>
<td>(14) ( \frac{-2a^2}{5b^2} ) or ( \frac{-2a^2}{5b^2} )</td>
<td>(5) ( 32 ) ( (c^2+d^2)(c-d)(c+d) )</td>
<td>(6)</td>
</tr>
<tr>
<td>(15) ( \frac{x(5x^2+10x+15)}{8} ) or ( \frac{5x^2+10x+15}{8} ) ( \text{or} ) ( \frac{5x^3+5x^2+3x}{8} ) or ( \frac{5x^2+5x^2+3x}{8} )</td>
<td>(8) ( 33 ) ( 2a(5x - 7)^2 ) or ( 2a(5x - 7)(5x - 7) )</td>
<td>(8)</td>
</tr>
<tr>
<td>( \frac{x(x-1)}{2(x-3)} ) or ( \frac{x^2-x}{2x-6} )</td>
<td>(9) ( 34 ) ( x = 3 ) and ( x = -\frac{9}{2} ) or ( -\frac{1}{2} ) ( \text{or} ) ( -4.5 )</td>
<td>(4)</td>
</tr>
<tr>
<td>( 2x^3-27x^2+36x ) or ( x(2x-3)(x-12) ) or ( x(2x^2-27x+36) )</td>
<td>(35) ( 4 - 2x + x^2 ) or ( x^2 - 2x + 4 )</td>
<td>(4)</td>
</tr>
<tr>
<td>( \frac{x+2y}{y(x+3y)} ) or ( \frac{x+2y}{xy+3y^3} )</td>
<td>(8) ( 36 ) ( x = 14 )</td>
<td>(4)</td>
</tr>
</tbody>
</table>

(Possible score for page, 96) (Possible score for page, 83)
Form A

Page 3

(37) \( \frac{x(2x + 5y)}{y(4x - 3y)} \) or \( \frac{2x^2 + 5xy}{4xy - 3y^2} \) (or)

(38) \( \frac{x - 3y}{x^2 + xy} \) or \( \frac{x + 3y}{x^2 + xy} \) (or)

(39) \( \frac{4a - 33}{a^2 + 3a - 28} \) or \( \frac{4a + 33}{a^2 + 3a - 28} \) (or)

Key

Page 4

(51) \( x = 1000 \) (3)

(52) \( x = \frac{1}{3} \) (5)

(53) A's = $2000 (3)

(54) L = 60 rd. (2)

(55) Eq: \( 7(x-4)+5(x-6) = 110 \) (3)

(56) Eq: \( \frac{x}{5} + \frac{y}{9} = 1 \) (4)

(57) $6300

(59) Prop: \( \frac{15}{x} = \frac{3}{10} \) (4)

or \( \frac{x}{10} = \frac{15}{3} \) or \( \frac{x}{15} = \frac{10}{3} \)

Men = 50

(60) \( .33(x) + .45(100-x) = .40(100) \) or

or \( 33x + 45(100-x) = 4000 \) (3)

or \( (33x + 45y) = 40(100) \) (3)

Oats = 412 \( \frac{2}{3} \) (3)

Corn = 58 \( \frac{1}{3} \) (3)

Possible score for page, 49 (Possible score for page, 72)
Form A

Page 5

Equation: \[5(n) + 10(n + 4) + 25(3n) = 670\]  
\[\text{No. of nickels: 7} \quad \text{(3 pts.)}\]
\[\text{No. of dimes: 11} \quad \text{(3 pts.)}\]
\[\text{No. of quarters: 21} \quad \text{(3 pts.)}\]

Amount at 6% $2140  
Amount at 6\frac{1}{2}% $2860

\[x = 3 \quad \text{(2 pts.)} \quad \text{(3 pts.)} \quad \text{(3 pts.)}\]
\[y = 4 \quad \text{(2 pts.)} \quad y = x + 1 \quad y = 2x - 2\]

<table>
<thead>
<tr>
<th>(x)</th>
<th>(y)</th>
<th>(x)</th>
<th>(y)</th>
</tr>
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<tbody>
<tr>
<td>-2</td>
<td>-1</td>
<td>-2</td>
<td>-6</td>
</tr>
<tr>
<td>-1</td>
<td>0</td>
<td>-1</td>
<td>-4</td>
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<td>0</td>
<td>1</td>
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<td>-2</td>
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<tr>
<td>1</td>
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<td>2</td>
<td>3</td>
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<tr>
<td>4</td>
<td>5</td>
<td>4</td>
<td>6</td>
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</tbody>
</table>

(3 pts. for each line)  
(Tables must be completely correct)

(Possible score for page: 39)
E. Answer Key for Form B
Indiana State—Horn—First Year Algebra Test
1934

Form B                  Key                        Total Possible Score: 340
(1) $x = -\frac{4}{5}$   - 8 (2) (20) $4x + 10 = 9x - 15$ or (3)  
(2) $x = -3$            3                           (3) (21) $T = \text{cu.ft. of bin}$  
(3) $x = .5$ or $\frac{1}{2}$ (4) (22) $S.P. = \text{Cost + Profit}$  
(4) $x = -\frac{1}{2}$  3                           (5) (23) $P = \frac{1}{9}$  
(5) $x = -6$            8                           (6) (24) $F = x^6 164$  
(6) $4x + 2y = 2(2x + y)$ (2) (25) $w = 10$  
(7) $-7a + 12b - 7c$  4                           (5) (26) $h = 8$  
(8) $\frac{5x^3 - 6x^2 + 3x - 1}{27x + 36}$ or (2) (27) $r = 6\%$  
(9) $\frac{3x^3 - 27x^2 + 36x}{x(2x - 3)(x - 12)}$ or (5) (28) $\left(\frac{x}{4} - \frac{1}{2}\right)\left(\frac{x}{4} + \frac{1}{2}\right)$  
(10) $49a^2 - 42ab + 9b^2$ (3) (29) $r(x - 8)(x - 4)$  
(11) $6a^2 - a^2 - 26a + 21$ (5) (30) $2y(7y - 2)(2y - 1)$  
(12) $4a^2 - ab + 2b^2$ (3) (31) $(a^2 + 4)(a - 2)(a + 3)$  
(13) $4x^2 - 2xy + y^2$ (4) (32) $(x - 2)^2(x + 2)^2$ or (6)  
(14) $4a$ (5) (x+2)(x-2)(x+2)(x-2)  
(15) $5a - 19$ (8) (33) $\frac{1}{5}(x - 3)(x + 1)$  
(16) $\frac{a}{a + b}$ (8) (34) $-5a^3$  
(17) $3a - 9b + 11c$ (9) (18) $\frac{1}{x(x - y)}$ or $\frac{1}{x^2 - xy}$ (9)  
(19) $\frac{a - 3}{a(a + 3)}$ or $\frac{a - 2}{a^2 + 3a}$ (9)

(Possible score for page 98)  
(Possible score for page 71)
Form B

Page 3

(35) \( \frac{2x^2 + x - 37}{6(x - 1)(x + 1)} \) or \( \frac{2x^2 - x - 37}{6x^2 - 6x} \)

(36) \( x = 238 \)

(37) \( x = \frac{2}{3} \) or \( 2 \)

(38) \( x = \frac{1}{3x - 1} \)

(39) \( x = 1\frac{1}{4} \)

(40) 3

(41) 4

(42) 3

(43) 4

(44) 2

(45) 1

(46) 3

(47) 2

(48) 3

(49) 4

(50) \( x = -\frac{1}{2} \)

(51) \( r = 4 \)

(52) \( a = 60 \)

(53) Price = $360

(54) \( L = 18 \text{ rd.} \)

(55) Short piece = 12.375

(56) Eq.: \( A = 2B + \$500 \)

\[ 6A = 9B + \$7200 \]

(57) Eq.: \( C + D = 116 \)

\( C = \pi D \)

(58) Prop. \( \frac{4.5}{2} = \frac{x}{352} \) or \( \frac{4.5}{2} = \frac{x}{352} \)

Building = 792 ft.

(59) Eq. \( (x = y + .06 \)

\[ 50(x + 6) + 70x = 3000 \]

\[ 50(x + .06) + 70x = 3000 \]

Cheaper kind = 22\(\frac{1}{4}\) ft.

Better kind = 28\(\frac{1}{8}\) ft.

(60) John = 80

Frank = 48

(Possible score for page, 51) (Possible score for page, 84)
Form B

Page 5

(81) Days worked = 55
Days idle = 5

(82) Men = 20

(83) \( x = -1 \)
\( y = -2 \)

(3 pts.)
\[ y - 2x = 0 \]

\[
\begin{array}{c|c}
 x & y \\
1 & 2 \\
0 & 0 \\
-1 & -2 \\
1 & 4 \\
-3 & -6 \\
\end{array}
\]

(3 pts. for each line)

\[ y + 2x + 4 = 0 \]

\[
\begin{array}{c|c}
 x & y \\
1 & -6 \\
0 & -4 \\
-1 & -2 \\
-2 & 0 \\
-3 & 2 \\
\end{array}
\]

(Possible score for page, 38)