AN EXPERIMENTAL COMPARISON OF TWO METHODS
OF TEACHING NINTH YEAR ALGEBRA

by

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Committee on thesis:

[Signatures]

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INTRODUCTION

High school mathematics, both in content and in method of teaching, has in the past few years undergone a storm of criticism. Among the contentions which have been advanced is that the large number of failures each year in high school algebra indicates either defective teaching methods or too rigid requirements in subject matter.

It has long been recognized that students learn at different rates and in different manners, and there has been a growing feeling that the traditional or daily recitation method of teaching does not provide ample opportunity for the differentiation of subject matter and instruction to suit the varying needs in any group of students.

In the light of this criticism it seemed a desirable project to ascertain by actual experimentation which of two outstanding methods of instruction in algebra would yield the better results if applied to a heterogeneous group in a small high school. It was felt that this thesis would have justified itself if it succeeded in accomplishing this purpose; and it was hoped also that it might give to teachers in the field a deeper interest in various methods of teaching and a little more inspiration to experiment, not by adhering to traditional material and method, but rather by teaching algebra as a mode of thought, as the basis of science and
engineering, as a means of understanding every day facts,' and as a device for character building.
CHAPTER I

THE EXPERIMENT AND DEFINITIONS OF TERMS USED

Statement of the experiment. It was the purpose of this experiment to determine whether the traditional recitation plan or the modified unit plan of instruction would yield greater returns when used with a heterogeneous group in a small high school; the results to be measured (1) "on the side of algebra, the ability to understand its language and use it intelligently, the ability to analyze a problem, to formulate it mathematically and to interpret the results,"\(^1\) (2) as a social subject, to make the world better by developing ideals of honesty, accuracy, industry, self-reliance, persistency, and an enthusiasm for knowledge.

Importance of the study. Algebra is recognized as a traditional subject in the secondary school. Probably because of this factor, few teachers of the subject have given evidence of real interest in exploring some of the newer concepts of educational methods. It is true that occasionally one finds teachers experimenting with new methods, but for

\(^1\)The Reorganization of Mathematics in Secondary Education. A Report of the National Committee on Mathematical Requirements. Under the auspices of The Mathematical Association of America Inc. 1923, p. 11.
the most part algebra remains a traditional subject taught in the traditional manner.

Effective teaching in any subject is dependent at least in a measure upon the use of effective methods. It is a common belief that the benefits which pupils derive from any course depend in part on the ability and the personality of the teacher, in part on the attitudes and abilities of the pupils, but to a larger extent on the teaching methods.

In this connection Charles H. Judd says:

The most promising subject in the curriculum can be turned into a formal and intellectually stagnant drill if it is presented by a teacher who has no breadth of outlook and no desire to teach pupils how to generalize experiences.

It is not far from the truth to assert that any subject taught with a view to training pupils in methods of generalization is highly useful as a source of mental training, and that any subject which emphasizes particular items of knowledge, and does not stimulate generalization is educationally barren.

With this broader view of teaching algebra in mind, and the adoption of new methods, the present storm against the subject may be diverted.

While the present day secondary school has reduced the required amount of mathematics to the minimum of one year in many places, and many elective courses have been abandoned

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because of lack of interest in the subject on the part of pupils, we are still cognizant of the fact that all pupils need some training in mathematical thinking. J. P. Everett, writing on "The Fundamental Skills of Algebra" says:

The pupil needs algebra for what it enables him to do but infinitely more does he need it for the orderly way in which in the midst of constant change, it enables him to think.

A subject of this importance should not be open to criticism or shunned as an elective subject if better methods of instruction will make it more effective and less distasteful to the students.

Certain units of algebra are required for college entrance, and because of that fact all pupils--particularly in our smaller schools--are required to take the subject despite the fact that the majority will not go to college. Fear of failure has brought about a general dislike of the subject, and this situation has brought us face to face with the fact that some experimentation is necessary if this attitude is to be changed. Writing on the subject, "The Gap Between Promise and Fulfillment in Ninth Grade Algebra", E. F. Lundquist said:

A significant proportion of high school pupils are by reason of mental ability, previous training, and

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3J. P. Everett, "The Fundamental Skills of Algebra", Teachers College Record, Columbia University. 1929, p. 3.
present motivation, incapable of deriving enough value from ninth grade algebra as it is being taught to justify its being required of all pupils.\textsuperscript{4}

"As it is being taught" seems to add emphasis to the idea of many educators that our present methods leave much to be desired. If there is to be improvement in our methods of instruction, more teachers in the field must be aroused to the experimental program necessary. There must be enrichment of teaching technique in order to create a revived interest, and greater accomplishment on the part of the pupil. More efficient schools should be the outcome of numerous studies of this kind.

Definitions and discussion of terms. The terms used in this thesis are "Recitation Method" and "Unit Method". By the recitation method we refer to that mode of teaching based on the Herbartian plan with which all teachers are familiar, namely: preparation, presentation, comparison, abstraction, generalization, and application. Morrison describes the ordinary recitation as follows:

Common practice is to treat either the contents of a book or a syllabus of a course of study as the learning product to be achieved...the process, by the action of problems, or by other practices which tend to stimulate reflection, but in the end learning is simply a process

of covering a given body of narrative, descriptive, or exposition discourse, or of solving a given series of problems.... The ground once covered is reviewed and the recitation form of testing the absorption of material is summarized in another test of the same sort and the same order, known as the examination.

Thayer gives us a brief history of the evolution of methods of teaching. He warns us that although the individual system of teaching is older than the recitation method it must not be confused with our present method.

It was spoken of by Grimshaw in 1855 as "the old-fashioned and false method of teaching individuals instead of classes". He pictured it thus: "I notice in my visits to the schools many pupils sitting idle; sometimes part of the school is asleep, or what is worse, making a noise and disturbing the remainder who desire to be industrious".

Thayer tells us that this system was replaced, a little more than a century ago, by a system of group teaching, which was originally devised by Joseph Lancaster. It was known as the monotorial system; and to it, Thayer tells us, we must give major credit for originating and developing the fundamental structure of the recitation. Of course, this structure has been greatly modified since that time; but the essential features are the same, and for such a procedure the activity and initiative are necessarily centered in the person of the teacher.


Now these ideas are in the process of change, and today a great deal is heard about the "child-centered school", and the teacher as "the guide". This has brought about the advancement of new methods of teaching to replace the traditional method. We must not lose the importance of the fact that no one acquainted with American education can seriously question the valuable contributions of the recitation method of instruction. Except for it...we should probably not have universal public education in this country. Consequently we should give all honor to the educational leaders who perfected it as a method of teaching and incorporated it as an integral part of their school organizations. They solved effectively the child educational problems of their day and laid the basis of future progress.

When we have given due credit to this view, we still have to face the problems peculiar to our day. We must ask the question, "Is the recitation method best adapted for present needs?" What is valuable at one stage of development is frequently injurious in another. It is always worth while to raise the question of the suitability of an old tool for the realization of new purposes. It is for this reason that we wish to inquire whether the traditional "lock-step" method of teaching, with its daily assignments of like tasks for all pupils, is adequately designed to realize present day educational aims and ideals.

Ibid., p. 12.
There have been a number of definitions given of the unit method. Byrne J. Horton, writing in *Educational Method*, tells us that Henry C. Morrison, professor of education in the University of Chicago, who is responsible for the first treatise on the unit plan defines it as, "...a comprehensive and significant aspect of the environment of an organized science, of an art, or of conduct, which being learned, results in an adaption in personality".8

W. H. Burton9 calls the unit of learning "an outcome". He further lists as the nature of units or outcomes: first, attitudes of understanding; second, attitudes of appreciation; third, special abilities. He states further that while many think the subject matter is the unit, it is really the assimilative material out of which the unit grows.

On the other hand, Reudiger10 of George Washington University defines a unit as "any division of subject matter, large or small, that when mastered gives one an insight into and appreciation of, or mastery of some aspect of life".

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8Byrne J. Horton, "A Brief Outline of the Unit Plan of Teaching," *Educational Method*, XII (December, 1932), pp. 75-77.


An editorial\textsuperscript{11} which reviewed the investigations of Roy O. Billett, of Illinois State Normal University and a member of the Staff of the National Survey of Secondary Education, reported that of all the plans to provide for individual differences, the unit plan is the most widely distributed and the most widely used.

Billett himself tells of his investigations.\textsuperscript{12} He says that the unit plan is a prominent characteristic of at least eleven plans in general use today, among which are the contract plan, the Dalton plan, the laboratory plan, the project method, the Morrison plan, and the Winnetka technique. He found that in actual practice these methods "differ in name only", irrespective of what the originators of these procedures may contend as to the differences represented. In actual practice, the procedures are practically identical.

The purposes of education are, in general, the purposes of the unit plan; namely, to attain "that stage at which the student has realized the meaning and purpose of study, has acquired the self-control which self-dependence implies, and has further acquired the range of methods of


\textsuperscript{12}Roy O. Billett, "Directed Learning and the Unit Assignment". Department of Secondary School Principals, Bulletin 45, March, 1933, pp. 55-76.
thinking and of study which removes him from constant dependence on the teacher". 13

The big objective in applying the unit plan to the teaching of algebra, according to E. Clarke Fontaine, 14 State Supervisor of High Schools in Maryland, is the understanding attitude leading the pupils to see why rather than teaching them to do mechanical tricks in manipulation of algebraic symbols.

With these objectives in mind, it is necessary to give particular attention to the teacher's part in carrying out this plan. The instructor is dealing with learners in groups and these individuals differ widely in all the factors of learning experience. According to Grinstead:

The teaching unit is merely a learning unit... in which the teacher assists the pupil in the attainment of his objectives. It is a cooperative enterprise in which the pupil... constitutes the center, the teacher functioning primarily as an expert adviser. The teacher's part is of no value whatever except as his help is effectively given. The goal, the learning activities, and the mastery are still the pupil's. 15

13Morrison, op. cit., pp. 211-212.

14E. Clarke Fontaine, "The Unit Plan of Teaching," Educational Administration and Supervision, XVIII (January, 1932).

Finally we will define the unit plan as used in this experiment as that plan devised by the teacher in which:

1. The classroom becomes the work room
2. The pupil receives a study sheet containing:
   a. Topic references
   b. Page references
   c. Explanations and explanatory references
   d. References for practice outside the text
   e. Problems to be worked from the text
3. The pupil works out his unit at his own rate, individually or in a group of his own choosing
4. The pupil's mastery of the subject is tested by unit tests, and by final tests provided by the state.

The teacher's place in the plan is to:

1. Maintain favorable conditions for study in the room
2. Enlarge upon the assignment
3. Stimulate interest
4. Direct and supervise the work
5. Give special attention to the slow and backward
6. Administer tests at the request of the student.

In short the plan is one in which the pupil takes the initiative rather than the teacher, as is the usual procedure in the traditional recitation.

So far as educators have expressed themselves, their approval seems unanimous for the unit method. While many different reasons have been given, the most emphasis has been on these facts: that first, it will care for individual differences; second, it will place the emphasis on the pupil rather than on the teacher; third, it will increase interest on the part of the pupil by allowing him to advance according to his ability without being compared too critically with others of greater ability or being retarded by others of less ability.
CHAPTER II

REVIEW OF PREVIOUS STUDIES

Although there has been much discussion of various methods of teaching, comparatively little experimentation has been done in the field of algebra. A few of the more recent experiments, similar in nature to this one, will be reviewed briefly.

Barton,¹ studying the effect of group activity and individual effort in developing the ability to solve problems, experimented with two groups of eleven pupils each, for the short period of twenty-eight days. His conclusions were that, for pupils of normal or above normal intelligence, the group discussion was superior.

Smith² experimented with two groups, based on age, sex, and I. Q. The control group received instruction and assignments daily, while the experimental group received mimeographed assignments covering work for several days. After thirty-two weeks experimentation, he concluded that the con-

¹W. A. Barton, "The Effect of Group Activity and Individual Effort in Developing Ability to Solve Problems in First Year Algebra," Educational Administration and Supervision, 12:512-519, November, 1926.

control group achieved more, judging from test results. He attributed the difference to administration difficulties, and to the teacher's lack of experience in individual instruction technique. This conclusion seemed to indicate that the results of the experiment were not completely convincing to the writer.

Mason measured the difference in results between contract assignments and recitation method in English, civics, American history, seventh grade arithmetic, biology, and first year algebra. He used two groups of eleven pupils each, pairing on the basis of I. Q. and Hotz algebra scales. He used the same scale at the end of the experiment as an end test of mastery. He reached the conclusion that pupils taught by the contract plan made greater achievement; the most consistent gain being made by the middle fifty per cent of the contract group.

Moore made a similar comparison and decided that neither method was significantly superior for the especially bright or slow pupil though he made no report as to the rel-

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ative value as it concerned the middle group.

Lawton\(^5\) reported the recitation plan better with less capable students, and the supervised study method better with able students.

Hunsiker\(^6\) found negligible differences in favor of larger unit assignments in algebra.

R. E. Gadske,\(^7\) on the other hand, using larger groups than any of the others, compared the unit system with the recitation system by testing with standardized tests and found a decided difference in favor of the individualized unit system. He used forty-six pupils equated on the basis of three standardized tests. The control group was taught by the lecture-demonstration method followed by tests and remedial instruction. The experimental group was taught by the unit method, individual assignments, and individual instruction. Achievement was measured by standardized tests. Mr. Gadske concluded:


\(^7\)R. E. Gadske, "A Comparison of Two Methods of Teaching First Year High School Algebra," School Science and Mathematics, XXXIII (June, 1933), pp. 635-640.
This conclusion is far from being a complete solution of all the relevant problems. It is simply one of many that must be made in order to reach a final conclusion. Nevertheless, it seems apparent that some importance may be attached to the findings, particularly since the individual unit method has turned out so superior to the traditional group method.

Burton J. Stallard and Harl Douglass made a rather extensive study of this nature in Wauwatosa, Wisconsin. They found a gain of 197 points as against 170 points in favor of the long unit groups. They discovered also, that the superiority of achievement of the exceptional group, from test to test, showed greater retention in those using the individual unit. In this experiment, the lower quartile of both groups had higher achievement according to ability than the upper quartile. Finally, they concluded in favor of the individual unit plan because it saved time for the teacher in explaining units; saved time for the gifted pupil; was adapted to any school without reorganization; and resulted in better relationship between teacher and pupil.

James V. Gainer failed to prove statistically that

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there was any difference between the two methods, but agreed with Stallard, Gadske, and others that there was enough promise in the experiment to warrant a great deal more experimentation along this line.

Marguerite Linn\textsuperscript{10} in her experiment took the broad view of the modern educator. In addition to the mastery of algebraic concepts, she studied the effect of the two methods on the character and attitudes of the pupils. As the measured test of achievement she used the results of standardized tests, and found in this little if any difference in the two methods. However, in her conclusions she stressed the fact that in the matter of attitudes and character building there was a decided difference in favor of the unit method.

This review of previous studies on the relative value of the unit plan and the recitation plan shows that there is much indecision. This very fact of indecision and disagreement found in reports of other experiments shows it to be an open field for further experimentation.

CHAPTER III

DESCRIPTION OF THE EXPERIMENT

This experiment was carried on with two groups of ninth year algebra students in the high school at Wingate, Indiana. Due to an administrative problem, these groups were maintained as the same heterogeneous groups which were promoted from the eighth grade. As a practical experiment this grouping was desirable because it represented the exact situation found in the small high school where the state requires all students graduating from a commissioned high school to have credit in one unit of mathematics. This requirement forces an unselected group into ninth grade algebra since the faculty in the small school is too limited to permit of an elective unit in mathematics. Heterogeneous grouping offers no disadvantage according to Luthur Purdom,¹ special educational investigator, who after investigating 442 high schools in Michigan, in algebra and English, reported that there was no significant advantage in homogeneous groups.

Data² accumulated at Palo Alto high school confirms


what educators have contended for some time, that the I. Q. as a determinant for grouping, is better than any other to predict probable success in algebra. With this in mind, during the first week of the term, Otis Self-Administering Intermediate test, Form B was given. The results obtained from this test were averaged with the results of Form A, which had been given at the end of the eighth year. On the basis of these averages, the eighteen pupils in each group were ranged. In the control group, known hereafter as Group A, the range was from 73 to 121. In the experimental group, known hereafter as Group B, the range was from 70 to 120. Following the method of Sorenson and Garrett, the former of whom wrote:

In most cases, the mean is the summary value generally accepted as the most accurate measure about which to calculate the measure of variability. In order to find the reliability of the difference between the two groups in terms of "chances that the obtained difference represents a true difference greater than zero," the formulas of Garrett were used. The mean I. Q. of Group A was 100.1 and of Group B,

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99.7. The reliability of the difference (.4) was found to be .02, which signified that the chances were 51 out of 100 that the true difference would be greater than zero. This is not considered a significant difference.\(^5\) Table I and graph I (pp. 19-20) show the I. Q. of each member of the groups and the graphical picture of their comparative abilities. It is to be noticed that though, in general, Group A has a slightly higher mental ability, yet the line of Group B drops below the control group at the lower end of the scale, indicating that this is the group to which special attention should be given.

During the first five weeks of the term both classes were taught by the recitation method. At the beginning of the sixth week, Group B began instruction by the unit method, while Group A continued with the recitation method. This plan was carried on throughout the remainder of the school year.

At the beginning of the experiment, the Hotz Algebra Scale, Form B, was given to each member of both groups as a base from which to measure accomplishment. (Table II, p. 21)

At the beginning of the experiment, the plan was explained to both groups. Group B then received the instruction

\(^5\)See Appendix.
**TABLE I**

INTELLIGENCE QUOTIENTS
OTIS SELF-ADMINISTERING TEST OF MENTAL ABILITY
INTERMEDIATE EXAMINATION--FORM A AND B

<table>
<thead>
<tr>
<th>GROUP A</th>
<th>GROUP B</th>
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<tbody>
<tr>
<td>Number</td>
<td>Otis A</td>
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<tr>
<td>1.</td>
<td>116</td>
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<td>2.</td>
<td>123</td>
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<td>3.</td>
<td>120</td>
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<td>4.</td>
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| | Median | 98.5 | | | Median | 105.5 |
| | Mean | 100.1 | | | Mean | 99.7 |
| | S. D. | 12.4 | | | S. D. | 15.2 |
| | Diff. Mean | .4 | | | Diff. Mean | .4 |
| | $\sigma_M.$ | 4.9 | | | $\sigma_M.$ | 6.8 |
| | $\sigma D$ | 8.48 | | | $\sigma D$ | 8.48 |
| | C. R. | .04 | | | C. R. | .04 |

C. R. .04 means that there are 51 chances in 100 that the true difference is more than zero.
GRAPH I

AVERAGE INTELLIGENCE QUOTIENTS
OTIS SELF-ADMINISTERING TEST OF MENTAL ABILITY
INTERMEDIATE EXAMINATIONS A AND B
TABLE II

FIRST SCORES ON HOTZ ALGEBRA SCALE FORM B

<table>
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<th>GROUP A</th>
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<td>17.</td>
<td>5</td>
</tr>
<tr>
<td>18.</td>
<td>4</td>
</tr>
</tbody>
</table>

* A--addition; S--subtraction; M--multiplication; D--division; Eq.--equations; F--formulae; Prob.--problems.

Mean 9.2 4.88 6.75 3.88 9.2 5.8 7.6 4.2
Diff. 0 1.08 .85 .3
S. D. 2.74 1.14 3.8 3.1 2.2 1.43 3.94 2.2
C. R. .34 .27 .8 .73 .52 .34 .93 .52

C. R. 2.6 means that there are 99.5 chances in 100 that the true difference is more than zero.

C. R. .69 means that there are 75.5 chances in 100 that the true difference is more than zero.

C. R. .33 means that there are 63 chances in 100 that the true difference is more than zero.
sheets containing:

1. Topic under consideration
2. Page references to the text
3. Explanation references to the text
4. Lists of problems to be solved
5. References to additional work on the same unit.

No definite number of problems was assigned, but it was explained that each one must master the unit and pass a satisfactory test before being allowed to proceed to the next unit. The tests were to be given when each individual was satisfied in his own mind that he had mastered the unit. The passing grade was 75%, but if the individual was not satisfied that he had done the best he could, he was given an opportunity to do more work on the unit and take a second test over the same unit. When each unit was completed satisfactorily, a new unit was given the pupil.

Since no definite time limit was placed on any unit, an outline of the units to be covered by the end of the semester was posted on the bulletin board. This plan was used so that each individual could check on his own progress and thus develop his ability to plan and be responsible for his own progress. During the time set for the recitation, the pupils worked individually or in groups of their own choosing, al-

---


7See Appendix for sample unit tests.
though it was noticed that comparatively little work was done in groups. No mention was made of work to be done outside of the classroom, that being left to the judgment of the pupil: another device leading to the development of purposefulness, the development of which is unquestionably an element in character building. The classroom became a work room in which there was always the hum of purposeful activity. The teacher assisted the pupils at their request, supervised the work, encouraged, and gave special attention to the slower and more timid pupils, who at first were somewhat reluctant to ask for help. At the request of the pupil the teacher administered a unit test.

Group A continued according to the traditional recitation method. With very little variation, the recitation program was as follows:

1. Definite assignment for the following day's work was made
2. Explanation of advance work was made by the teacher, using the explanations in the book and illustrating with supplementary problems from Betz and Hart 8, 9
3. Correction of papers prepared on the previous assignment
4. Explanation of difficulties of the previous lesson


5. Class drill on the previous lesson either by short tests or work on the board

6. Work on the advance assignment by the pupils and teacher, the class forming the rules of procedure to be used. These were put in simple form by the pupils so that everyone could understand them and use them in preparing the advance assignment.

The same units were used in this section as in Group B. At the end of each unit the class was given the same test as had been used in the other section. The results of these tests were kept as a measure of progress by the teacher in order to keep in close touch with what each pupil was doing. The results of the unit tests were not used however in the final measure of accomplishment because they had not been administered in the same manner to both groups.

At the end of each semester, Manchester tests\(^{10}\) were used as a measure of accomplishment. These tests were used because they are the tests used over a large part of the state as a standard measure of ninth year algebra.

At the end of the year, both groups were given, as a final test, the test which had been used by Indiana University in the 1937 State Sectional Algebra Contest.\(^{11}\) As a further test of accomplishment Hotz Scale, Form B, was again given,\(^{12}\) in order to measure accurately the gain made since

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\(^{10}\)See Appendix.

\(^{11}\)See Appendix.

\(^{12}\)See Appendix.
the beginning of the experiment.

A very definite attempt was made to hold the variables constant. The same teacher conducted both classes, both classes used the same text, and both recited in the morning; section A reciting the first period, and section B reciting the second period. The same time was devoted to the recitation. The same tests were given as a measure of final achievement, the testing material covering only the first thirteen units which had been covered by all members of both groups. All the tests were time limited to fifty minutes except the final test which was ninety minutes in length. All of the final tests were given in the same room and at the same time. Although the unit tests were given in such a way as to avoid as much as possible the probability of one member of the group who had taken the test early giving aid to another, the results of these tests have not been used in the final comparisons. In giving these tests, paper was supplied on which all the work was to be shown. All paper was handed in at the end of the test and no papers were returned until the end of the experiment.
CHAPTER IV

RESULTS OF THE EXPERIMENT

After performing an experiment such as this, the logical question arises, "What are the results?" Of course it is impossible to measure, by means of any test yet devised, some of the most important values resulting from such a study; but one thing which can be measured definitely, and accurately, is achievement in the mastery of the fundamentals of subject matter.

If the intelligence quotient is a true predictive measure of probable success, these groups will be equally successful in their mastery of the fundamental concepts of algebra, unless the difference in the method of instruction gives one group an advantage over the other. In order to determine whether or not there is any difference, the results must be compared critically and according to scientific method.

Success was measured at the end of the first semester by the first semester Manchester Test,¹ and the results tabulated in Table III.² The expected median of this test, as

²Table III, p. 27.
### TABLE III

**FIRST SEMESTER SCORES**  
**MANCHESTER TEST**

<table>
<thead>
<tr>
<th>Number</th>
<th>Group A</th>
<th>Group B</th>
</tr>
</thead>
<tbody>
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<td>98</td>
</tr>
<tr>
<td>2</td>
<td>88</td>
<td>97</td>
</tr>
<tr>
<td>3</td>
<td>83</td>
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<td>5</td>
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<td>6</td>
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<td>7</td>
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<tr>
<td>18</td>
<td>30</td>
<td>46</td>
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</tbody>
</table>

<table>
<thead>
<tr>
<th></th>
<th>Median</th>
<th>Mean</th>
<th>Diff.</th>
<th>S. D.</th>
<th>S. M.</th>
<th>S. D.</th>
<th>C. R.</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>73.5</td>
<td>70.6</td>
<td>8.2</td>
<td>16.5</td>
<td>3.9</td>
<td>3.96</td>
<td>1.49</td>
</tr>
<tr>
<td></td>
<td>83.5</td>
<td>78.8</td>
<td>8.2</td>
<td>16.7</td>
<td>3.96</td>
<td>5.5</td>
<td></td>
</tr>
</tbody>
</table>

C. R. 1.49 means that there are 93.5 chances in 100 that the true difference is more than zero in favor of Group B.
indicated by the makers of the test, was 68. It is noticed
that both groups exceeded this median; Group A by 5.5 points
and Group B by 15.5 points, showing a ten point better median
for the class doing work by the unit method. Comparing the
mean or average of the two groups, the experimental group
exceeded the control group by 8.2 points. When the reliabil-
ity of this difference was computed,\(^3\) it was found to be 1.49
which, according to Garrett's table,\(^4\) means that there are
93.5 chances in 100 that the difference between the true
measures is greater than zero. Unless the value of 3 is
found, this ratio is not considered statistically as a sig-
nificant difference.

Since this group is small, it is advantageous to study
the results by means of graphical representation. Graph II
(p. 29) shows that with but two exceptions the members of
Group B equalled or exceeded the members of Group A. Partic-
ular attention is called to pupil 16 of Group B, who though
having an I. Q. lower than pupils 13, 14, 15, 16, and 17, of
Group A exceeded them in the semester test. Attention is also
called to pupil 18 of Group B, who having an I. Q. of only 70

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\(^3\)H. E. Garrett, *Statistics in Psychology and Education*
(New York: Longmans, Green and Co., 1926), pp. 118-134. See
Appendix for formulas used in computation.

\(^4\)Ibid., p. 134.
GRAPH II
FIRST SEMESTER SCORES
MANCHESTER TEST
exceeded the lowest member of Group A by several points. This graph would seem to indicate that, on the whole, the unit method gives better results and particularly with those of lower mentality.

At the end of the second semester, since Group B had covered more units than Group A, the Second Semester Manchester test was carefully checked to see that no problems were included which had not been included in the units completed by both groups. The results of this test were tabulated in Table IV. The tabulations showed a median of 69.5 for Group A and 73 for Group B, as against an expected median of 56, which was designated by the makers of the test. Again both classes exceeded expectations by 13.5 points and 17 points respectively—a margin of gain of 3.5 in favor of the experimental group. At this time there was a difference of means of only 3.1 in favor of Group B. The reliability of this difference was .46 indicating 67 chances in 100 that the true difference was more than zero. Statistically this is not a significant difference.

The graphical representation shows a very wide variation in grades in both classes, which probably accounts for

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5Table IV, p. 31.
6Graph III, p. 32.
### TABLE IV

SECOND SEMESTER SCORES
MANCHESTER TESTS

<table>
<thead>
<tr>
<th>Number</th>
<th>Group A</th>
<th>Group B</th>
</tr>
</thead>
<tbody>
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<tr>
<td>2.</td>
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<tr>
<td>3.</td>
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<td>88</td>
</tr>
<tr>
<td>4.</td>
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<td>94</td>
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<tr>
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</tr>
<tr>
<td>6.</td>
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<td>56</td>
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<td>69</td>
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<tr>
<td>8.</td>
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<td>89</td>
</tr>
<tr>
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<td>73</td>
</tr>
<tr>
<td>16.</td>
<td>39</td>
<td>55</td>
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<tr>
<td>17.</td>
<td>44</td>
<td>27</td>
</tr>
<tr>
<td>18.</td>
<td>23</td>
<td>19</td>
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</table>

<table>
<thead>
<tr>
<th></th>
<th>Group A</th>
<th>Group B</th>
</tr>
</thead>
<tbody>
<tr>
<td>Median</td>
<td>69.5</td>
<td>73</td>
</tr>
<tr>
<td>Mean</td>
<td>65.7</td>
<td>68.8</td>
</tr>
<tr>
<td>Diff.</td>
<td>3.1</td>
<td></td>
</tr>
<tr>
<td>S. D.</td>
<td>18.3</td>
<td>21.4</td>
</tr>
<tr>
<td>σ M.</td>
<td>4.3</td>
<td>5.04</td>
</tr>
<tr>
<td>σ D.</td>
<td></td>
<td>6.6</td>
</tr>
<tr>
<td>C. R.</td>
<td>.46</td>
<td></td>
</tr>
</tbody>
</table>

C. R. .46 means that there are 67 chances in 100 that the true difference is more than zero in favor of Group B. This is not a significant difference.
GRAPH III
SECOND SEMESTER SCORES
MANCHESTER TEST
the small statistical difference. Particular attention is called to pupil 16 of Group B, who again exceeded pupils 13, 14, 16, and 17 of Group A. The graph of this semester's grades shows less significant results than before. However, it appears, both from the table and the graph, that as the work progresses in difficulty, there is less advantage to be found in the unit method.

Before tabulating the results of the State Sectional Algebra Contest examination it was checked carefully to see that all problems contained in it were included in the units covered by both classes. The results of this test were tabulated in Table V.7 There was a difference in the medians of 29.5 points, and a difference in mean of 18.6 points in favor of the experimental group. Statistically the critical ratio between the groups is 2.14, which indicated 98.3 chances in 100 that the true difference is greater than zero. This is not considered a significant difference; yet it is worthy of note that on an examination testing the total mastery of the subject, the statistical difference is very close to a significant difference in favor of Group B.

The graphical representation8 of these two groups of

7Table V, p. 34.
8Graph IV, p. 35.
<table>
<thead>
<tr>
<th>Number</th>
<th>Group A</th>
<th>Group B</th>
</tr>
</thead>
<tbody>
<tr>
<td>1.</td>
<td>107</td>
<td>127</td>
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<td>2.</td>
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<td>124</td>
</tr>
<tr>
<td>4.</td>
<td>120</td>
<td>113</td>
</tr>
<tr>
<td>5.</td>
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</tr>
<tr>
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<td>82</td>
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<tr>
<td>8.</td>
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</tr>
<tr>
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<td>85</td>
</tr>
<tr>
<td>12.</td>
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<td>88</td>
</tr>
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<tr>
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<td>60</td>
</tr>
<tr>
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<td>52</td>
</tr>
<tr>
<td>18.</td>
<td>35</td>
<td>42</td>
</tr>
</tbody>
</table>

Median: 61 | 90.5
Mean: 71.7 | 90.3
Diff.: 18.6
S. D.: 26.6 | 25.7
σ M.: 6.2 | 6.1
σ D.: 8.7
C. R.: 2.14

C. R. 2.14 means that there are 98.3 chances in 100 that the true difference is more than zero in favor of Group B.
grades shows but two points where a member of Group B drops below the line of Group A. Study of the graph gives evidence that Group B, as a whole, has attained a higher degree of mastery in algebra than Group A; particularly is this fact evident in the case of the lower half of the class.

The results of the first Hotz test, given at the beginning of the experiment, are found in Table II (p. 21). Study of this table indicates that, at the beginning of the experiment, Group B was superior in multiplication and division, by a mean difference of 1.8; in simple equations and formulae, by .8; and in problem solving, by .3; while in addition and subtraction the groups were the same. In the same test, administered at the end of the experiment, there was a difference in favor of the experimental group in each division of the test. These differences, however, were in no case great enough to prove, by statistical measurement, any significant difference in the groups.

Comparison of the results in the two Hotz scales showed a total gain of 313 points for Group A, and 398 points for Group B—a difference of 85 points in favor of those instructed by the unit method. There was a difference of 4.7

9Table VI, p. 37.

10Table VII, p. 38.
C. R. 1.3 means that there are 91 chances in 100 that the true difference is more than zero in favor of Group B.

C. R. .88 means that there are 81 chances in 100 that the true difference is more than zero in favor of Group B.

C. R. 1.8 means that there are 96 chances in 100 that the true difference is more than zero in favor of Group B.

C. R. .7 means that there are 76 chances in 100 that the true difference is more than zero in favor of Group B.
<table>
<thead>
<tr>
<th></th>
<th>GROUP A</th>
<th>GROUP B</th>
<th></th>
<th>GROUP A</th>
<th>GROUP B</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>Number</td>
<td>First</td>
<td>Second</td>
<td>Gain</td>
<td>Number</td>
</tr>
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<td>1</td>
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<tr>
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<td>18.</td>
<td>7</td>
<td>11</td>
<td>4</td>
<td>18.</td>
<td>9</td>
</tr>
</tbody>
</table>

Total gain 313 398
Mean 17.4 22.1
Diff. 4.7
S. D. 5.49 7.15
\( \sigma \) M. 1.29 1.92
\( \sigma \) D. 2.13
C. R. 2.04

C. R. 2.04 means that there are 97 chances in 100 that the true difference is more than zero in favor of Group B.
points in the means, and the reliability of this difference was found to be 2.04, signifying 97 chances in 100 that there is a true difference greater than zero in favor of the unit method. This scale, like the State Contest examination, is a comprehensive test covering an entire years work. The statistical result on each of these tests was nearer a significant difference than in the case of either of the semester tests. This would indicate that the unit plan gives better results when retention and comprehension of the entire years work are considered.

The graphical representation\footnote{Graph V, p. 40.} shows but four points where a member of Group A equalled or excelled the corresponding member of Group B. In only one instance, that of number 17, where the difference was 8 points, was there a difference of more than two points.

For a more critical comparison, the experimenter selected eleven pairs of pupils whose I. Q. rating differed by no more than four points.\footnote{Table VIII, p. 41.} With this group we notice that though the gains in the comprehensive tests did not reach a statistical difference, the difference is much greater than in either of the semester tests. The average of
GRAPH V

COMPARATIVE GAINS HOTZ TEST

[Graph showing comparative gains over class numbers with two lines, one black and one red, indicating different groups.]
TABLE VIII

SEMESTER AVERAGES, STATE CONTEST EXAMINATION, AND HOTZ GAINS WITH 11 PUPILS PAIRED WITH NO DIFFERENCE GREATER THAN 4

<table>
<thead>
<tr>
<th>Num-</th>
<th>Semester Final</th>
<th>Hotz</th>
<th>I. Q. Average</th>
<th>Test Gain</th>
<th>Num-</th>
<th>Semester Final</th>
<th>Hotz</th>
<th>I. Q. Average</th>
<th>Test Gain</th>
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<tr>
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<td>1.</td>
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<td>1.</td>
<td>120 93.5</td>
<td>127 30</td>
<td>120 93.5</td>
<td>127 30</td>
</tr>
<tr>
<td>2.</td>
<td>117 94</td>
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<td>2.</td>
<td>117 96.5</td>
<td>137 35</td>
<td>117 96.5</td>
<td>137 35</td>
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<tr>
<td>3.</td>
<td>117 81.5</td>
<td>87 21</td>
<td>3.</td>
<td>114 90.5</td>
<td>3.</td>
<td>114 90.5</td>
<td>124 29</td>
<td>114 90.5</td>
<td>124 29</td>
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<td>4.</td>
<td>113 83</td>
<td>120 20</td>
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<td>112 95</td>
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<td>112 95</td>
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<td>112 95</td>
<td>113 27</td>
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<td>109 71.5</td>
<td>60 16</td>
<td>5.</td>
<td>110 78</td>
<td>5.</td>
<td>110 78</td>
<td>108 24</td>
<td>110 78</td>
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<td>6.</td>
<td>106 73.5</td>
<td>94 22</td>
<td>6.</td>
<td>106 90</td>
<td>6.</td>
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<td>105 21</td>
<td>106 90</td>
<td>105 21</td>
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<tr>
<td>7.</td>
<td>102 49.5</td>
<td>73 13</td>
<td>7.</td>
<td>106 78</td>
<td>7.</td>
<td>106 78</td>
<td>93  21</td>
<td>106 78</td>
<td>93  21</td>
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<tr>
<td>8.</td>
<td>94  63.5</td>
<td>50 16</td>
<td>8.</td>
<td>96  70</td>
<td>8.</td>
<td>96  70</td>
<td>77  17</td>
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<td>77  17</td>
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<tr>
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<td>94  57</td>
<td>47 17</td>
<td>9.</td>
<td>90  73.5</td>
<td>9.</td>
<td>90  73.5</td>
<td>72  23</td>
<td>90  73.5</td>
<td>72  23</td>
</tr>
<tr>
<td>10.</td>
<td>91  67</td>
<td>52 20</td>
<td>10.</td>
<td>87  70.5</td>
<td>10.</td>
<td>87  70.5</td>
<td>63  20</td>
<td>87  70.5</td>
<td>63  20</td>
</tr>
<tr>
<td>11.</td>
<td>73  26.5</td>
<td>35 4</td>
<td>11.</td>
<td>70  32.5</td>
<td>11.</td>
<td>70  32.5</td>
<td>42  9</td>
<td>70  32.5</td>
<td>42  9</td>
</tr>
</tbody>
</table>

| Median | 105.5 70.3 | 73.5 18 | 108 84 | 100.5 22.5 |
| Mean   | 103.5 68.2 | 77.3 17 | 102.5 75.4 | 91.4 23.3 |
| Diff.  | 13.6 18.5 | 29.8 5.4 | 14.4 21.6 | 27.9 6.7 |
| S. D.  | 4.1  5.6  | 8.9  1.64 | 4.3  6.5  | 8.4  2.02 |
| σ D    | 5.9  8.5  | 12.2  2.6 |
| C. R.  | 0.34 .64  | 1.15 2.42 |

C. R. .34 means that there are 63 chances in 100 that the difference is more than zero in favor of Group B.

C. R. .64 means that there are 73.8 chances in 100 that the difference is more than zero in favor of Group B.

C. R. 1.15 means that there are 87 chances in 100 that the difference is more than zero in favor of Group B.

C. R. 2.42 means that there are 99.2 chances in 100 that the difference is more than zero in favor of Group B.
the two semester grades, the final grades, and the gains on the Hotz tests are represented graphically in pairs in Graphs VI, VII, and VIII\textsuperscript{13} to show comparative accomplishment. In reading these graphs it is readily seen that in the semester averages and in the final averages, the members of Group B made the greater gain. In the case of the gains on the Hotz test, with this pairing, there was but one pair in which there was a small difference in favor of a Group A member.

Since the age of these two groups has not yet been taken into consideration, a glance at Table IX\textsuperscript{14} and at Graphs IX and X\textsuperscript{15} will show that, as a group, the control group is 7.1 months older than Group B, and mentally, 5 months older. The graphs show that at the lower end of the mental scale the members of Group B are younger than those of similar rank in Group A. Some school authorities consider age of sufficient significance to recommend that freshmen of low mentality wait until their second high school year to take algebra in the belief that the added year will in a measure offset the low mentality. This has been done in the Laboratory School, Indiana State Teachers College. The re-

\textsuperscript{13}Graphs VI, VII, VIII, pp. 43, 44, 45.
\textsuperscript{14}Table IX, p. 46.
\textsuperscript{15}Graphs IX, X, pp. 47, 48.
GRAPH VIII

GAINS ON HOTZ TEST
ELEVEN PAIRS PUPILS: I. Q. WITHIN FOUR POINTS
<table>
<thead>
<tr>
<th>Number</th>
<th>Chronological Age</th>
<th>Mental Age</th>
<th>Number</th>
<th>Chronological Age</th>
<th>Mental Age</th>
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<td>197</td>
<td>2.</td>
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<td>5.</td>
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<td>197</td>
<td>184</td>
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<td>184</td>
<td>172</td>
<td>9.</td>
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<td>12.</td>
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<td>166</td>
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<td>177</td>
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<td>14.</td>
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<tr>
<td>15.</td>
<td>201</td>
<td>162</td>
<td>15.</td>
<td>186</td>
<td>146</td>
</tr>
<tr>
<td>16.</td>
<td>185</td>
<td>152</td>
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<td>146</td>
<td>17.</td>
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<td>124</td>
</tr>
<tr>
<td>18.</td>
<td>189</td>
<td>108</td>
<td>18.</td>
<td>205</td>
<td>127</td>
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<table>
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<th></th>
<th>Mean</th>
<th>185.4</th>
<th>176.6</th>
<th>178.3</th>
<th>171.6</th>
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<tbody>
<tr>
<td>Diff</td>
<td>7.1</td>
<td>5.</td>
<td></td>
<td></td>
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</tr>
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</table>
GRAPH IX

CHRONOLOGICAL AGES OF GROUPS A AND B
GRAPH X

MENTAL AGES OF GROUPS A AND B
results of this experiment do not coincide with the belief that older children do better in algebra for here we found the younger children have made greater gains as a whole. Particularly is this true of those of low mentality in Group B, who are younger than the low I.Q.s of Group A, and who have made noticeable gains over those in the recitation group.

Because the testing was done only over work which had been completed by all members of both groups, we have made no mention of the comparative amounts of work covered. Group A completed thirteen units of work while Group B completed eighteen units—the entire text with the exception of the chapter on Numerical Trigonometry. This fact is definitely on the side of the unit instruction plan.

So far as the first of the objectives—subject mastery—is concerned, the limited scope of this experiment shows little difference in the groups as measured by statistical methods. Viewing the results from the point of graphical representation, there can be no question but that the group using the unit plan of instruction made greater gains, and attained better mastery of the subject.

No final conclusion should be drawn without considering the second factor: the study as a social subject in which it was the aim to test the attitude of the pupils toward the acquisition of knowledge, and the development of ideals of
honesty, accuracy, industry, self-reliance, and persistency.
The measurement of attitude is hard except as the pupils
themselves express their opinions. In order to ascertain the
attitude of the pupils in an unbiased manner, the cooperation
of the English department was enlisted. The teacher in that
department asked the pupils in her classes to write a theme
in which they expressed their opinion of the two methods being
used in the algebra classes. She assured them that the con-
tent of their themes would not be divulged to anyone until
after the final grades had been published. In this way she
was unusually successful in getting some very original and
frank opinions.

The general consensus of opinion was in favor of the
unit plan. Every one in the experimental section except two
favored the plan. One of these said he did not like it be-
cause "there was too much to do". He further gave as a reason
for not liking the plan that he always lost his head on an ex-
amination and did not do as well as he thought he should. In
the class using the recitation plan there were seven who ex-
pressed a desire to try the unit plan because it sounded in-
teresting. This general favoring of the unit plan agrees with
the results of an investigation of Billett. He compared the
reactions of 263 pupils to the unit plan as contrasted with
classroom procedures, after six months experience with the
unit plan. He says:

One must conclude that boys and girls of all levels of academic intelligence, accomplishment, and application, whether the subject matter be geometry or English, regard the unit plan as a distinct improvement over the traditional recitation procedure.

Some excerpts from the papers of the students seem to furnish the best proof that their attitude toward the subject and the plan has made the experiment of value as a social subject. These excerpts, quoted from the pupils' papers, are of more value as evidence than the subjective opinion of the writer or of any teacher.

I like the new plan because you learn to do things for yourself and be independent. If something happens like being sick or going to a party, so you don't get to work your problems, you don't get a zero. You work them the next day to make up. (B)

I like the units plan because it gives us a better chance to get a good grade because we have to get all the problems to pass the test. Sometimes when I don't have to, I only work a few of them. I like it, too, because there is not as much chance to cheat when we take the examinations alone. The members of the other class can cheat on the examination easier. Sometimes they don't get all the problems but wait until class and have the teacher explain them. In our class you have to get them all to learn how to do them on the examination. (B)

I like the new plan because the teacher is not always hounding me to get my problems. It looks as if she thought we had more sense when she leaves it up to us to get our lessons. If we don't get them we can't blame her for she is ready to help us when we need it. (B)

I like the new plan because the teacher comes around to see how we are getting along and helps us. (B)

I like the units because I am afraid to recite and I am so slow at the board that the rest always have to wait for me and they call me "slow poke". I don't get as good grades as the others but nobody knows it but the teacher and me. (B)

I don't know whether I would like the unit plan or not but they all seem to like it because they work on algebra a lot when they have their other lessons. Maybe if I had that plan I would work more and not read so much. I wish I had been in the other class. (A)

I don't know whether I would like the unit plan or not but they all seem to like it because they work on algebra a lot when they have their other lessons. Maybe if I had that plan I would work more and not read so much. I wish I had been in the other class. (A)

I like our class better than I would the other class because I know just what I have to do every day. If I had to do it the way the others do I would never get my lessons. Sometimes I don't anyway. (A)

I like the recitations because I like to go to the board and race the others to see who can get done first. I don't think the other way would be any fun. (A)

I wish I were in the other class. If I were I could go just as fast as I want to. As it is, I get all my work and have to sit around with nothing to do while she explains to some of those who do not understand. Some of them haven't even tried. I think I could do as much as Edwin if I had a chance. I think we should have been asked which we wanted to do. (A)

Regarding the third important factor, that of character building, the conclusions must of necessity be purely subjective. Of the eight other teachers who had observed these pupils, either in class or in study hall, all but one were of the opinion that the unit plan had helped the pupils to become more independent in their thinking and in their actions. They were unanimous in their opinion that the pupils had learned how to discriminate between what they knew and what
they needed help to find out. All the teachers agreed that the pupils, who were working under the unit plan, were more industrious. The one who disapproved of the plan did so because he said that the pupils in Group B spent too much time on algebra and neglected other subjects. When questioned specifically about the results as observed in the less gifted pupils, they were of the opinion that the less gifted were more sure of themselves, and less timid about asking for help.

If the unit method will develop in the student independence of thought and action, initiative, intellectual honesty, and confidence in one's self, these are contributions which are, without doubt, of value in educating citizens. Other things being equal, if the unit plan does no more than aid in character building, it has a very definite place in the field of mathematics.
CHAPTER V

CONCLUSION

This experiment has certain shortcomings which cannot be overlooked. The small number of pupils, equated as groups rather than as individuals; and the wide range of variability due to heterogeneous grouping (although giving a life situation) made for unreliability in statistical measurement. The opinion of the writer on this point is at variance with that of H. R. Shibles, who after experimenting with twenty pupils, wrote in his conclusion:

With these groups there was not a wide range of intelligence quotients and hence the difference between them is small. A heterogeneous group or one with a wide range would have been more valuable.1

In spite of these shortcomings, the findings in this experiment are so consistently in favor of the unit plan that the writer is inclined to draw tentative conclusions pending further experimentation along this same line.

The reliability of the difference in the first semester Manchester test was 1.49, indicating that there were 93.5 chances in 100 that the true difference in favor of Group B was more than zero; the second semester Manchester test scores showed a reliability of .46, indicating 67 chances in

100; and the state contest examination revealed reliability of 2.14 or 98 chances in 100 that the difference was more than zero in favor of Group B.

Comparison between the two Hotz tests showed that Group A made 313 points gain and Group B, 398 points. The reliability of this difference was 2.04, indicating 97 chances in 100 in favor of Group B.

In the smaller group of twenty-two students equated in pairs with no greater difference than four points in I. Q. rating, findings showed consistently better results in Group B. The reliability of the difference in average semester grades was .64, indicating 73.8 chances in 100 in favor of Group B; state contest grades showed 1.15 reliability, indicating 87 chances in 100; and gains on Hotz scale were 2.24, indicating 99.2 chances in 100 in favor of Group B. It is also to be noted that in every case but two the members of Group B made greater gains than those of Group A.

Chronologically and mentally, Group A exceeded Group B, but in spite of this advantage Group B exceeded Group A in every point of comparison.

Evaluating the groups on the bases of attitude toward the subject and development of good citizenship traits, the balance is decidedly in favor of the experimental group.

The unit plan of instruction is not an easy plan to
put in operation. The teacher's work is increased a great deal in preparing the units, in keeping the records, and in conducting the recitation hour. In spite of the increased labor, and the small difference, as shown statistically, between the recitation and the unit methods, the writer is convinced that the unit plan is best fitted to care for the individual differences in the ninth grade algebra classes. For the teacher who is genuinely interested in teaching pupils rather than subject matter, it holds immeasurable possibilities. Pupil and teacher are brought closer together, and an opportunity is created for better development of personality traits along lines of good citizenship; and at the same time the pupil is given a better mastery of the concepts of ninth grade algebra.

It cannot be said conclusively that the unit plan is the best in all situations and under all circumstances; for, so long as education is dealing with individual personalities, we will never be able to use the same standard of measurement in all situations. However, the writer is convinced that the unit plan gave results enough better than those produced by the recitation plan to commend it to the consideration of the teacher who is trying to work out the method best qualified to truly educate youth. It is perhaps not too much to hope that the unit plan may serve as a means of giving a partial answer to the criticism against algebra as a required subject; and therefore to make the future of mathematics secure in our secondary schools.
BIBLIOGRAPHY
BIBLIOGRAPHY

A. BOOKS


B. PERIODICAL ARTICLES


Everett, J. F., "The Fundamental Skills of Algebra," Teachers College Record, Columbia University, 1929. p. 3


E. PUBLICATIONS OF LEARNED ORGANIZATIONS

Report of National Committee on Mathematical Requirements, under auspices of Mathematical Association of America, Inc., 1923, p. 11.

G. UNPUBLISHED MATERIALS


ABBREVIATIONS AND FORMULAS USED IN TEXT

S. D............. Standard deviation

σ M............. Standard Error of the Mean

Diff............. Difference between Means

σ D............. Standard Error of the Difference

C. R............. Critical Ratio

F................. Frequency

D................. Deviation

\[
S. D. = \sqrt{\frac{F D^2}{n}}
\]

\[
σ M = \frac{S D}{\sqrt{n}}
\]

\[
σ D = \sqrt{σ M_1^2 + σ M_2^2}
\]

C. R. = \frac{M_1 - M_2}{σ D}

C. R. must be 3 to be considered a significant difference.

Table—Garrett, p. 134.

UNIT ONE


I. Addition of Polynomials
   1. See example p. 47
   2. Exercise 52
   3. Study carefully the method of checking p. 53
   4. Exercise 54
      a. Check each problem carefully
         (notice problem 7)

II. Subtraction of Polynomials
   1. Study example 55
      a. Define terms subtrahend, minuend, difference
      b. Make a rule for subtraction
   2. Exercise 56
      a. Read the problem carefully to notice which is
         the subtrahend

III. Multiplication
   1. What is the sign of the product when numbers with
      like signs are multiplied? Unlike signs?
   2. Exercises pp. 52, 53 through problem 21
   3. What is the purpose of the parentheses?
      a. What does 5(2a-3) mean?
   4. What relation do multiplication and division bear to
      each other? What then will be the rule for signs in
      division?

IV. Have you made a satisfactory grade on each exercise?

V. Problems of this type are to be found in Wells, 1934
   edition, p. 93, 102
   Betz, p. 203, 205
   Nyberg, p. 69-72

VI. If you are satisfied you may ask for the unit test.
UNIT TWO

Linear Equations pp. 54-66.

I. Solve $3x = 12$
   1. What does the term $3x$ mean?
   2. If $3$ times some number you do not know equals $12$, what is the value of the unknown number? How did you get it? What fundamental process did you use?

II. $-5x = 20$
   1. What is unusual about this equation?
   2. What rule must you keep in mind?
   3. What is your answer?

III. In this same manner solve problems 1-15, ex. 62.

IV. What is an equation? What do you call the parts of an equation? What different things do you think you could do to both members of an equation without changing its value? See ex. 63, 64, 65

V. Exercises 66, 67, 68, 69.

VI. Look up the meaning of the word transpose. Study articles 70, 71.

VII. What has been done in each step in the following?

   $12 + 8x = 5x - 3$
   $8x - 5x = 12 - 3$
   $3x = -15$
   $x = -5$
   $12 - 40 = -25 - 3$
   $-28 = -28$

VIII. Solve and check ex. 73, p. 59 and 60 through problem 42

IX. Write equations for problems 43-55

X. Parentheses
   $3(a + b)$ is read "three times the quantity $a + b$"
   1. Exercises 77-80

XI. Have you graded your exercises? Are you ready for your test?
UNIT THREE

Formula

I. What is a symbol?
Write symbols for the following words.

1. length 11. volume 21. cost
2. width 12. area 22. multiply by
3. height 13. diagonal 23. Fahrenheit
4. altitude 14. distance 24. number
5. base 15. time 25. plus
6. perimeter 16. velocity 26. times
7. diameter 17. rate 27. minus
8. radius 18. interest 28. equals
9. pi 19. profit 29. percent
10. surface 20. loss 30. weight

II. Write in symbols
1. Rate times time
2. Gain plus loss
3. Base times altitude
4. Length plus width
5. Two times pi times radius
6. Circumference divided by pi
7. Pi times diameter
8. Distance divided by rate
9. A number (n) minus three
10. Principal times rate
11. Volume divided by height
12. Perimeter minus twice length
13. Profit multiplied by two
14. Area divided by length
15. Amount minus principal
16. 9/5 times centigrade (temperature) plus 32 degrees
17. Fahrenheit is equal to zero
18. Seven per cent of a number
19. Rate times time equals distance
20. Velocity times the time

III. Express the principle of each of the following formulas in a good English sentence:

1. A = lw
2. A = S^2
3. V = Bh
4. I = prt
5. S = Se^2
6. A = 1/2bh
7. C = 2πr
8. A = πr^2
9. A = p + i
10. V = e^3
11. d = rt
12. S = 2πrh
IV. What measurements do you make to find the area of a circle? The volume of a cylinder? Surface of a cube? Area of a triangle?

V. Exercises 81 and 82 in your book.
Problem solving

You are now ready to put the symbols that you have been writing into problems. You will find that by the use of symbols will make the solving of problems much easier.

I. How would you write in symbols one number increased by 3?

II. Write one number decreased by 7.

III. There are two numbers. One is three more than the other.
Write expressions for both numbers.
Let $x$ = the smaller number
then $(x+3)$ = the larger number.
Now we will indicate the addition of these numbers as:
$x + (x+3)$
Now we will try a problem—there are two numbers. One of the numbers is four more than the other. Their sum is ten. What are the numbers?
Let $x$ = the smaller
$(x+4)$ = the larger
$x + (x+4) = 10$
$x + x + 4 = 10$
$2x = 10 - 4$
$2x = 6$
$x = 3$

IV. Study exercises 88, 89, 90 very carefully.

V. Exercise 91.
The purpose of this exercise is to translate statements into equations.
Look back on page 20 for the form we used in simple equations at the first of the term.
Read each problem carefully, then with your book closed see if you can tell it in your own words. Ask yourself the following questions:
What am I asked to find?
What did the problem tell me?
What shall I let $x$ stand for? Why?
Can I get expressions for the other unknown numbers?
Is there a statement that will form an equation?
Solve problems and check all results.
Minimum work on this unit Grade D
Exercises 91, 92, 93, 94.

Maximum work pp. 75-104.

Are you ready to try the test?
UNIT FIVE

Equations Containing Fractions pp. 105-124.

I. Translate the statements into algebraic expressions.
   1. If the width of a field is expressed by the term \( x \), how would you express \( \frac{1}{2} \) the width? \( \frac{1}{4} \) the width? \( \frac{3}{4} \) the width?
   2. If the length of a field is expressed by \( (x+3) \) how would you express \( \frac{1}{2} \) the length? \( \frac{3}{4} \) the length?

II. Translate the problems in exercise 109.

III. Multiplication of fractional expressions. Ex. 110.

IV. Solution of equations containing fractions.
   1. Previous equations have contained no fractions. Notice principle IV, p. 108.
   2. What number could you use to multiply by in order to get rid of every denominator? Have you used the smallest number that would do?
   3. If your denominators were 3, 4, and 6, what number would you choose, 12 or 24? Why?

V. Procedure
   1. Select multiplier
   2. Multiply every term. (Why every term?)
   3. Solve by
      a. Transposing
      b. Collecting terms
      c. Finding value of the unknown
   4. Check (Substitute in the original equation) Why not use the equation in which you have cleared of fractions?
UNIT ONE TEST

I. Add the following
1. \(6x -7y + 3z\)
   \(-2x + 3y + z\)
   \(x -2y - z\)

2. \(2a -3b -5c\)
   \(-5a + b -3c\)
   \(3z - b + 9c\)

3. \(-2a^3 + a^2 + 5a -6; \quad 6a^3 -2a + 3; \quad 4a^3 + 8a^2\)

4. \(2A -3B + 5C; \quad 3A -6C; \quad -5A -B + C\)

5. \(6a + b + 7c; \quad -3a -6b + 4c; \quad 5a + 4b -6c\)

Add the following polynomials and then check the results by letting \(a=2, \ b=3, \ x=4, \ y=5\)

6. \(3x -6y + 3a; \quad -5x + 2y -5a; \quad 7x + y -6a\)

7. \(5a -2b + 3y; \quad 6a + 4b -4y; \quad -3a -b + 2y\)

Fill blanks with proper words

8. To ________ one number from another change the ________ of the ________ and then apply the rules for the ________ of numbers.

9. The quotient of two numbers with ________ signs is ________

10. The quotient of two numbers with ________ signs is ________

11. Plus times ________ equals plus

12. Minus times ________ equals plus

13. ________ times minus equals minus

14. Minus times plus equals ________

15. To add two or more numbers with unlike signs ________ the figures and ________ the answer the ________ of the ________

16. To add two or more numbers with like signs ________ the figures and prefix the sign of the ________ to the result

17. When you multiply expressions with exponents you ________ the exponents

18. When you divide you ________ the exponents

19. (a) \(2(-3x + 5y)\)  \(y^2(y^4 -y^2)\)

20. \(-4(5x + 3y)\)  \(x^3(x^2-x)\)

21. \(-5(x(x^2-x))\)  \(5(-a+b)\)

22. \(-6x^2(3x -2)\)  \(3(6x -7y)\)
UNIT TWO TEST

I. Solve the following
1. \(-5x = 75\)
2. \(3x = -8\)
3. \(-5x = -15\)
4. \(3x = 5\)
5. \(5x = 5\)

II. Solve
1. \(5x - 8 = 32\)
2. \(4x - 19 = -3\)
3. \(6x = 56 - 2x\)
4. \(7x = 50 + 2x\)
5. \(6x = 3 - 17 + 2x\)

III. Solve and check
1. \(5x - 2 + 3x = 14\)
2. \(-3y + 6 = 15 - 5y\)
3. \(x - 5 + 2x = 70\)

IV. Write as equations the following:
1. The sum of \(x\) and 6 is 13
2. 35 equals \(x\) increased by 12
3. Three times a certain number equals 18 added to the number
4. Twice a certain number equals 11 subtracted from the number
5. If 5 is subtracted from 12 times a certain number, the result equals 15 more than 8 times the number.

V. Simplify:
1. \(4(5x - 3) - 2\)
2. \(6(-2x + 3) - 5\)
3. \(8(3 - 4x) - 7\)
4. \(5(3a - 2) + 4(3a + 1)\)
5. \(2(5 - 3x) - 6(x + 4)\)

VI. Simplify:
1. \((6x - 2) - (-3x + 4)\)
2. \(4 - 5b - (2 - 3b)\)
3. \(x - (x - 3y) + y\)
4. \(2x - y - (x + y)\)
5. \(5x - (3x + 2) + 4x\)

VII. Solve:
6. \((x - 3) = 5(x - 1)\)

VIII. Solve:
2. \(2x = 9 - 3(x - 2)\)

IX. \(1/2(8x - 4) = 1/3(6x - 9)\)

X. If \(r = 3, \pi = 3.17, h = 10,\) find the value of \(A = 2\pi r (r + h)\)
UNIT THREE TEST

1. Write a formula for the following rules:
   a. The perimeter of a rectangle is equal to twice the length plus twice the width
   b. The distance traveled is equal to the time, times the rate
   c. The area of a square equals the square of the side
   d. \( \pi \) is equal to the quotient of the circumference divided by the diameter
   e. The amount is equal to the principal plus the interest
   f. Area is equal to half the sum of the three bases times the height
   g. Volume is equal to \( \pi \) times the square of the radius times the height.

2. Write formula for each of the following statements:
   a. The cost of \( N \) footballs at \( d \) dollars each
   b. The height of a building of 5 stories if each story is \( m \) feet
   c. An even number if \( n \) represents any odd number
   d. The number of inches in \( f \) feet and 4 inches
   e. The number of cents in \( r \) dollars
   f. The number of games won if a team played \( n \) games and lost 10
   g. The savings if a man earned \( D \) dollars and spent \( x \) dollars.
UNIT FOUR TEST

1. There are 46 pupils in a room, and the number of boys is 12 more than the number of girls. How many boys and how many girls are there in the room?

2. A board 31 feet long is divided into three pieces so that the second is 6 feet longer than the first, and the third is 3 times the length of the first. What is the length of each piece?

3. In an isosceles triangle, two of the angles are always equal. If the third angle of an isosceles triangle is 40 degrees more than one of the others, find the size of each angle.

4. The sum of three numbers is 30. The second is 12 more than the first, and the third is 4 times the first. Find the numbers.

5. A family spends twice as much for food as for rent, and $200 a year less for clothing than for rent. If these expenses total $3000, how much is spent for each item?

6. The capacity of one freight car is 4 tons more than that of another. A locomotive is pulling 10 of the smaller cars and 5 of the larger cars, making a total haul of 620 tons. What is the capacity of each kind of car?

7. One transport carried 400 more soldiers than another. In 5 trips of the larger and 3 trips of the smaller, 19,600 soldiers were carried. How many men did each ship carry?

8. A is 12 years older than B. Find the ages of each if 9 times A's age added to 4 times B's age equals 368 years.

9. John has 12 more nickels than dimes. How many nickels and how many dimes has he if he has $1.50?

10. A goes six miles an hour and B goes nine miles an hour. A starts two hours before B. How many hours has B been traveling when they are 102 miles apart?
UNIT FIVE TEST

Write expressions for the following:
1. 1/3 of x
2. 2/5 of the sum of x and 10
3. 2/3 of the sum of x and y
4. 2/5 of a certain number decreased by 10

5. A is x years old. B is 6 years younger. What is 1/5 of B's age?
6. Multiply 40(7 - 3x)
7. 20(x - 3/4)

8. 15 - \(\frac{x + 1}{3}\)
9. -12 \(\frac{-5x}{4}\)
10. 16(5x - 6)

11. \(\frac{x - 12}{3}\)
12. \(\frac{5x - 3}{6}\)

13. \(\frac{x + 2}{6} + \frac{x}{3} = \frac{x - 4}{9}\)
14. \(\frac{x - x - 13}{6} = \frac{2}{12}\)

15. The sum of two numbers is 60. The sum of 3/4 of the smaller and 2/5 of the larger is 31. Find the two numbers.

16. The width of a rectangle is 3/5 of the length. Find the dimensions if the perimeter is 160 feet.

17. A can do a job in 12 days and B in 15 days. After A worked 5 days, B came to help and worked until the work was completed. How long did B work?
18. Solve \(\frac{9x + 1}{5} - \frac{7}{15} = x\)
19. \(\frac{6x + 1}{3} - \frac{1 - 3x}{4} = 1\)

20. State in words the meaning of \(\frac{a + b}{2}\); \(\frac{a - 2}{b}\); \(ab + 2\);

\(a + bc\); \(a(b+c)\).
Date of Test........................................... Total Score......................................

MANCHESTER SEMESTER-END HIGH SCHOOL TESTS

FIRST YEAR ALGEBRA TEST
First Semester

NAME.................................................................................................................................

Home Address.....................................................................................................................

School....................................................... Teacher..................................................

Directions: You should have several sharpened pencils. If you break your pencil points, raise your hand and the teacher will supply you with another.

First of all, fill in the above blanks. Write your name clearly and plainly. Do not turn the page over until the signal to begin is given. After you are told to "go," turn the page and read the instructions at the beginning of the first exercise. Do your best to follow the directions carefully. Ask no questions after the signal to begin is given. Continue doing your best throughout the entire test until after you reach the end of the test, unless the signal to stop is given before you have completed the test. The time limit is 60 minutes.

Directions for Scoring: The highest possible score is 100 for all of the Manchester Semester-end High School Tests. How much each item counts in scoring is indicated in the number of the item. Most of the items count one point; but some of the items in this test count more than one point. For example, if an item is numbered 70-73, it counts four points; if it is numbered 70, it counts only one point. Therefore, the way the item is numbered indicates the number of points each item should count in scoring this test.

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for \( x_1 = 3x - 5 = (5 + 3x) \).
FIRST YEAR HIGH SCHOOL ALGEBRA

First Semester

Part I—Best Answer Exercise

Directions: Complete the following algebraic expressions by writing the number of the best answer in the parentheses on the right of each item.

Example: If \( x = 2 \), then the value of \( 2x \) is (1) 8 (2) 4 (3) 6 (4) 9 (2)

1. If \( b=8 \); \( h=4 \), the value of \( \frac{h}{b} \) is (1) 1 (2) \( \frac{1}{2} \) (3) 2 (4) 32 (___)

2. \( h^2 \) means (1) \( h \cdot h \) (2) \( 2h \) (3) \( \frac{1}{2}h \) (4) \( h \) (___)

3. When you add \( 6x^2+4+3x^2+2 \), you get (1) \( 3x^2+2 \) (2) \( 15x^2 \) (3) \( 9x^2+6 \) (4) \( 9x^2+6 \) (___)

4. The equation for the following statement: "Three times a certain number equals 20" is (1) \( 3n = 20 \) (2) \( n = 20x3 \) (3) \( 20n = 3 \) (4) \( 3n+20 = 0 \) (___)

5. If one angle of a triangle is 40° and another 60° the third is (1) 10° (2) 80° (3) 90° (4) 100° (___)

6. If \( x = -3 \), \( y = -2 \), \( a = -5 \) then the value of \( \frac{5xy}{a} \) is equal to (1) 6 (2) -7.5 (3) -6 (4) -3 (___)

7. By adding \( 6x+3a-4x+7a \) the result is (1) \( 12ax \) (2) \( 10x-10a \) (3) \( 20a-x \) (4) \( 2x+10a \) (___)

8. By division \( \frac{-15x}{5} \) simplifies to (1) \( 3x \) (2) \(-3x \) (3) \( -3 \) (4) \(-3x \) (___)

9. The sum of \( 4x \) and \( 5y \) is (1) \( 9xy \) (2) \( 4x+y \) (3) \( x+y \) (4) \( 4x+5y \) (___)

10. The equation for "one angle of a triangle is 4 times as large as another angle; the third angle is 7 times the smaller number is" is (1) \( 4n+7n = 180 \) (2) \( n+4n+7n = 180 \) (3) \( n+4n+7n = 90 \) (4) \( n+4n = 7n \) (___)

11. The sum of \( 2a-3b+7c \) and \( 7a-9b-8c \) is (1) \( 9a-12b-c \) (2) \( 5a-6b-15c \) (3) \(-5a+6b-c \) (4) \(-20abc \) (___)

12. When you subtract \( 4a+3b \) from \( 5a+4b \) the difference is (1) \( 9a+7b \) (2) \(-a-b \) (3) \( a+b \) (4) \( 16ab \) (___)
13. By multiplying $-4x$ by $-3x^4$ the product is (1) $12x^4$ (2) $-12x^4$ (3) $12x^5$ (4) $-12x$ (5) $3x^4$ (6) $-3x^5$ (7) $3x^5$ (8) $-3x^4$ (9) $-3x^4$ (10) $-3x^5$ (11) $-3x^4$

14. By solving $2x = -8$, $x$ will equal (1) $-4$ (2) $4$ (3) $16$ (4) $-14$

15. By removing parentheses from $2x(3x-4)$ the product is (1) $6x^2-4$ (2) $3x-8x$ (3) $2x^2$ (4) $6x^2-8x$

16. The formula for finding the area of a circle is (1) $C = 2\pi r$ (2) $A = \frac{1}{2} bh$ (3) $A = \pi r^2$ (4) $A = bh$

17. If $a^2$ is divided by $a^3$ the quotient is (1) $a^{-1}$ (2) $a$ (3) $a^2$ (4) $a^{10}$

18. The equation for finding interest is (1) $A = P + Pr$ (2) $i = Prt$ (3) $i = \frac{Fr}{t}$ (4) $i = \frac{Fr}{t}$

19. If one ton of coal occupies 38 cu. ft., the formula for finding the number of tons $T$ of coal that a bin will hold is (1) $T = \frac{LWH}{38}$ (2) $T = 38 LWH$ (3) $T = \frac{WH}{38L}$ (4) $T = \frac{LWH}{38}$

20. The equation for “A can do a certain piece of work in 12 days and B can do it in 8 days. If they work together how many days are needed?” is (1) $\frac{8}{x} + \frac{12}{x} = 1$ (2) $\frac{x}{8} = \frac{x}{12}$ (3) $\frac{x}{8} - \frac{x}{12} = 1$ (4) $\frac{x}{8} + \frac{x}{12} = 1$

21. In solving $y-3x=4$, $y$ equals (1) $4-3x$ (2) $3x+4$ (3) $\frac{3x}{4}$ (4) $\frac{4}{3x}$

Part II—Completion

Directions: Below are statements with a word left out. You are to choose the word that will best complete the thought. Place the number of that word in the parentheses at the right.

Example: Algebra is a study in __________________. (1) English (2) history (3) mathematics (4) home economics (5) English (6) history (7) mathematics (8) home economics (9) English (10) history (11) mathematics (12) home economics (13) English (14) history (15) mathematics (16) home economics

22. A rule expressed by literal numbers and mathematical symbols is called a __________________. (1) triangle (2) formula (3) square (4) fraction

23. The 2 in $36h^2$ is called the __________________. (1) coefficient (2) literal number (3) decimal (4) exponent

Solve for $x$: $3x - 8 = 5 + 2x$
24. Such terms as $4x$ and $3x$ are called .......... terms because in each term the same number $x$ has been multiplied by some other number (1) like (2) unlike (3) solution (4) equation

25. If one of the angles of a triangle is a right angle the triangle is called a .......... triangle. (1) protractor (2) isosceles (3) right (4) equilateral

26. The product of two numbers having like signs is .......... (1) negative (2) unlike (3) positive (4) opposite

27. The result of division is called the .......... (1) sum (2) product (3) difference (4) quotient

28. A single term, such as $5x$, is called a .......... (1) polynomial (2) trinomial (3) monomial (4) coefficient

29. If the sum of two angles is $180^\circ$ they are (1) complementary (2) equilateral (3) supplementary (4) isosceles

30. In graphs, the two numbers which locate a point are called .......... of the point (1) ordinates (2) axis (3) co-ordinates (4) abscissa

31. If one member of an equation is divided by any number the other member must be .......... by the same number. (1) added (2) subtracted (3) multiplied (4) divided

32. Transposing is the same as .......... the opposite of any term to both members of an equation. (1) adding (2) subtracting (3) multiplying (4) dividing

33. When numbers are compared by division the relation between the two numbers is called their .......... (1) proportion (2) ratio (3) sum (4) difference

34. In any .......... the product of the extremes equals the product of the means. (1) ratio (2) proportion (3) equation (4) problem

35. In graphs, the point where the axes cross is called the .......... (1) coordinate (2) abscissa (3) ordinate (4) origin

36. In solving two equations by the multiplication-addition method or substitution method the pair of numbers that satisfies both equations is called the .......... of the two equations. (1) solution (2) graph (3) multipliers (4) method
Part III—Matching Exercises

Directions: In the parentheses below, write the numbers of those left hand items which best match the right hand items. Not all left hand items will be needed.

Group 1

1. $8x^3$

2. $4x^3$

3. $\frac{1}{4}$

4. $8x^6$

5. $2$

37. $(\frac{1}{2})^2$ - - - - - (........)

38. $(2x^3)^3$ - - - - - (........)

39. $(2x^3)(2x^3)$ - - - - - (........)

Group 2

1. $\frac{x-4}{x}$

2. $4-x$

3. $x-4$

4. $7x^3-7xy$

5. $7xy-7x^3$

40. $-1(4-x)$ - - - - - (........)

41. $-7x(x^2-y)$ - - - - - (........)

42. $-\frac{4-x}{x}$ - - - - - (........)

Group 3

1. $x = 7$

2. $x = y$

3. $x = -y$

4. $x = 52$

5. $x = 8$

43. $6x + 5 = 47$ - - - - - (........)

44. $x - y = 0$ - - - - - (........)

45. $\frac{x}{2} = 4$ - - - - - (........)

Group 4

1. $x^2y^5$

2. $-5$

3. $x^2y^3$

4. $3a-5$

5. $5$

46. $\frac{x^5y^4}{x^3y}$ - - - - - (........)

47. $\frac{-30a^3b^5}{-6a^3b^5}$ - - - - - (........)

48. $(6a^2-10a) \div 2a$ - - - - - (........)

14. Solve for $x$: $-3x = 8 + (5 + 2x)$
Part IV—Graphs

Directions: On the squared space below draw on the set of axes the two graphs. (One point for each point located correctly, and two points for correct graph solution of the two equations represented by the graphs.)

<table>
<thead>
<tr>
<th>x</th>
<th>y</th>
</tr>
</thead>
<tbody>
<tr>
<td>49.</td>
<td>(-3)</td>
</tr>
<tr>
<td>50.</td>
<td>(-1)</td>
</tr>
<tr>
<td>51.</td>
<td>1</td>
</tr>
<tr>
<td>52.</td>
<td>3</td>
</tr>
<tr>
<td>53.</td>
<td>5</td>
</tr>
<tr>
<td>54.</td>
<td>(-3)</td>
</tr>
<tr>
<td>55.</td>
<td>(-1)</td>
</tr>
<tr>
<td>56.</td>
<td>1</td>
</tr>
<tr>
<td>57.</td>
<td>3</td>
</tr>
<tr>
<td>58.</td>
<td>5</td>
</tr>
</tbody>
</table>

59-60. At what point do the graphs cross? (Place answers in parentheses.)
Part V—Equations to Solve

Directions: Solve following equations and place answers in parentheses.

61-63. (3 points) \( 6x - 3 = 17 + 2x \) 
\( x = \ldots \)

64-66. (3 points) \( 2 - (3x - 4) = 6x - 12 \) 
\( x = \ldots \)

67-70. (4 points) \( \frac{x}{2} - 12 = \frac{x}{3} \) 
\( x = \ldots \)

71-74. (4 points) \( \frac{2x - 9}{3} = \frac{x + 11}{5} \) 
\( x = \ldots \)

75-78. (4 points) \( \frac{y + 4}{3} - \frac{y + 12}{6} = \frac{y - 2}{12} \) 
\( x = \ldots \)

Part VI—Story Problems

Directions: Solve the problems below in the space provided after each one. Each correct answer counts points as indicated at each problem.

79-81. (3 points) One number is twice another. Their sum is 30. What is the smaller number? 
\( \ldots \)

82-84. (3 points) One angle of a triangle is 2 times as large as the smallest, and the third angle equals the sum of the other two. How many degrees are there in each of the three angles? 
First Angle \( \ldots \) 
Second Angle \( \ldots \) 
Third Angle \( \ldots \)
85-88. (4 points) An investment of $6000, part of it at 5% and the remainder at 8%, yields an income of $375. How much is the amount invested at each rate?

- - - - At 5% (.............)
- - - - At 8% (.............)

89-92. (4 points) A is 12 years older than B. Find the age of each if 9 times A's age added to 4 times B's age equals 368 years?

- - - - A's Age (.............)
- - - - B's Age (.............)

93-96. (4 points) John has 12 more nickels than dimes. How many nickels and how many dimes has he if he has $1.50?

- - - - Nickels (.............)
- - - - Dimes (.............)

97-100. (4 points) One number exceeds another by 39. Find the numbers if their sum is 67.

- - Smaller Number (.............)
- - Larger Number (.............)
Directions: You should have several sharpened pencils. If you break your pencil points, raise your hand and the teacher will supply you with another pencil.

First of all fill in the above blanks. Here and throughout the test write clearly and plainly. Do not turn over the page until the signal has been given. Then turn the page, read the instructions carefully, and work thoughtfully through the exercises and questions until you have finished or until the signal to stop is given. The time limit is 60 minutes. Follow directions carefully, and do not ask any questions after you have started.

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FIRST YEAR HIGH SCHOOL ALGEBRA

Second Semester

Part I—Best Answer Exercise

Directions: Complete the following algebraic expressions by writing the number of the best answer in the parentheses on the right of each item.

Example: The value of $a^2 \cdot a^3$ is (1) $a^6$ (2) $a^{8/3}$ (3) $a^5$ (4) $a^{2/3}$

1. The product of $-2xy \cdot -5x^2y$ is (1) $-10x^3y^2$ (2) $10x^2y^2$ (3) $10x^4y$ (4) $-10x^2y$

2. The product of $c^6 \cdot c^3 \cdot c^4$ is (1) $c^{13}$ (2) $c^{60}$ (3) $3c^{12}$ (4) $3c^{60}$

3. $10^2 \cdot 10^3$ is equal to (1) $100^5$ (2) $100^6$ (3) $10^6$ (4) $10^5$

4. A simple way of writing $(x^3)^4$ is (1) $x^{12}$ (2) $x^7$ (3) $4x^3$ (4) $3x^4$

5. The indicated product of $a(a^2-3a+5)$ is (1) $a^2-3a+5$ (2) $a^3-3a+5$ (3) $a^2+3a^2+5a$ (4) $8a^2$

6. The product of $(x+2)(x-3)$ is (1) $x^2+x-6$ (2) $x^2-5x+6$ (3) $x^2-x-6$ (4) $x^2+5x+6$

7. The product of $(2x-7)(x-1)$ is (1) $2x^2-9x+7$ (2) $x^2-7x-7$ (3) $2x^2-5x-7$ (4) $2x^2-9x-1$

8. $(a+3)^2$ expanded, or written out in full is (1) $a^2+9$ (2) $a^2+6a+9$ (3) $a^2-6a-9$ (4) $a^2-9$

9. The factors of $x^2-25$ are (1) $(x-5)$ $(x-5)$ (2) $(x-5)$ $(x+5)$ (3) $(x-9)$ $(x+1)$ (4) $(x-3)^2$

10. The factors $x^2+6x+9$ are (1) $(x+3)^2$ (2) $(x-3)$ $(x+3)$ (3) $(x-9)$ $(x+1)$ (4) $(x-3)^2$

11. $x^4 y^5$ divided by $x^2 y$ is equal to (1) $x^2 y^6$ (2) $x^2 y^4$ (3) $x^2 y^5$ (4) $xy^8$

12. The factors of $x^2-x$ are (1) $x(x-1)$ (2) $(x-1)$ $(x-1)$ (3) $(x-1)$ $(x+1)$ (4) $x^2$ $(x-1)$

13. The factors of $x^2+2x-80$ are (1) $(x-20)$ $(x+y)$ (2) $x(2x-80)$ (3) $(x+10)$ $(x-8)$ (4) $(x-10)$ $(x+8)$

14. $-7x^2y$ divided by $-7x^2y$ is equal to (1) 1 (2) 0 (3) $xy$ (4) $-1$

15. $\frac{15xy}{10x}$ can be written (1) $2xy$ (2) $150x^2y$ (3) $\frac{2y}{5}$ (4) $\frac{3y}{2}$
6. \((2x+7) (x+5y)\) is divided by \(x+5y\) the result is
   (1) 0 \hspace{1cm} (2) \(x+7\) \hspace{1cm} (3) \(2x+7\) \hspace{1cm} (4) 1

7. \(\frac{a^2-9}{a-3}\) simplified is
   (1) \(a+3\) \hspace{1cm} (2) \(\frac{a+3}{a-3}\) \hspace{1cm} (3) \(a-3\) \hspace{1cm} (4) \(\frac{1}{2}\)

8. \(\frac{2}{a} + \frac{3}{b}\) added together equals
   (1) \(-\frac{5}{a+b}\) \hspace{1cm} (2) \(\frac{2b+3a}{ab}\) \hspace{1cm} (3) \(\frac{6}{ab}\) \hspace{1cm} (4) \(\frac{2b+3a}{a+b}\)

9. In the equation \(a+2x = b\); \(x\) equals
   (1) \(a(a+b)\) \hspace{1cm} (2) \(\frac{a-b}{2}\) \hspace{1cm} (3) \(\frac{ab}{2}\) \hspace{1cm} (4) \(\frac{b-a}{2}\)

10. \(3x+ax-bx\) added together equals
    (1) \(3abx^2\) \hspace{1cm} (2) \(x(3+a-b)\) \hspace{1cm} (3) \(3(3ab)\) \hspace{1cm} (4) 0

11. Solving \(bx-a = b\) for \(x\), \(x\) will equal
    (1) \(a\) \hspace{1cm} (2) \(\frac{b-a}{b}\) \hspace{1cm} (3) \(\frac{a+b}{b}\) \hspace{1cm} (4) \(-a\)

---

**Part II—Completion**

**Directions:** Below are statements with a word left out. You are to choose the word that best completes the thought. Place the number of that word in the parentheses at the right.

**Example:** A rule expressed by literal numbers and mathematical symbols is called a (1) triangle (2) formula (3) square (4) fraction. 

2. A factor which consists of just a simple term, like the term \(2a\), is called (1) binomial (2) monomial (3) square (4) double.

3. If the product of two or more factors is zero, then at least one of the factors must be (1) one (2) zero (3) quotient (4) sum.

4. The kind of fraction we get when we change \(\frac{5}{3} \div \frac{8}{3}\) is (1) proper (2) mixed (3) improper (4) binomial.

5. A fraction is not changed in value if both the numerator and (1) _____ are multiplied by the same number (2) multiplier (3) denominator (4) fraction.

6. To find the L.C.M. of several quantities, factor each one of the quantities and find an expression which is (1) divisible (2) multiplied (3) factored (4) subtracted.

7. A fraction whose numerator or denominator contains fractions is (1) simple (2) complex (3) compound (4) convenient.
28. We may change the sign of both the numerator and denominator ... changing the sign of the fraction (1) without (2) by (3) when (4) merely

29. If s is the sum of two numbers and d their difference we always know that the larger of the two numbers is always one-half of the ....?.... of s and d (1) difference (2) quotient (3) product (4) sum

30. The sign \[\sqrt{\text{?}}\] is called a ....?.... sign (1) division (2) multiplication (3) radical (4) adding

31. Radicals like 2 \[\sqrt{3}\] and 5 \[\sqrt{3}\] are called like radicals because the ....?.... is or are the same (1) root (2) radicands (3) coefficients (4) product

32. A right triangle is one in which one of the angles is ....?.... angle (1) a right (2) an obtuse (3) an acute (4) an equal

33. The law for multiplication of terms having exponents is that you ....?.... the exponents (1) subtract (2) divide (3) multiply (4) add

34. For a fractional exponent the ....?.... indicates the root (1) denominator (2) numerator (3) exponent (4) coefficient

35. In a distance problem the rate times the time always gives ....?.... (1) hours (2) distance (3) time (4) rate

36. 24 = 3 . 2 . 2 . 2 The 3. 2 . 2 . 2 are called the ....?.... factors of 24 (1) prime (2) monomial (3) binomial (4) fractional

Part III—Matching Exercises

Directions: In the parentheses below, write the numbers of those left-hand items which best match the right-hand items. NOT ALL LEFT HAND ITEMS WILL BE NEEDED.

**Group 1—Factoring**

<table>
<thead>
<tr>
<th>(1) (x-6) (x+4)</th>
<th>(2) (x-12) (x+2)</th>
<th>(3) (x-4) (x+3)</th>
<th>(4) (x-4) (x-3)</th>
<th>(5) (x+3) (x+4)</th>
</tr>
</thead>
<tbody>
<tr>
<td>37. x^2-x-12</td>
<td>38. x^2-2x-24</td>
<td>39. x^2-7x+12</td>
<td></td>
<td></td>
</tr>
</tbody>
</table>

**Group 2—Factoring**

<table>
<thead>
<tr>
<th>(1) (x-25) (x+1)</th>
<th>(2) (x-5) (x-5)</th>
<th>(3) (x+25) (x+1)</th>
<th>(4) (x-5) (x+5)</th>
<th>(5) (x+5) (x+5)</th>
</tr>
</thead>
<tbody>
<tr>
<td>40. x^2-25</td>
<td>41. x^2-10x+25</td>
<td>42. x^2-24x-25</td>
<td></td>
<td></td>
</tr>
</tbody>
</table>
Group 3—Square of Binomial

1. \( a^2 + 2ab + b^2 \)
2. \( 9a^2 - 6ab + b^2 \)
3. \( 4a^2 + 8ab + 4b^2 \)
4. \( a^2 - 2ab + b^2 \)
5. \( 9a^2 - 4a^2 - b^2 \)

Group 4—Literal Equations to Solve

1. \( x = a - 4 \)
2. \( x = 4 - a \)
3. \( x = 4a \)
4. \( x = a/4 \)
5. \( x = 4/a \)

Group 5—Addition of Like Radicals

1. \( \sqrt{16} \)
2. \( 10 \sqrt{3} + \sqrt{6} \)
3. \( 2 \sqrt{6} \)
4. \( 12 \sqrt{3} \)
5. \( \sqrt{5} + \sqrt{6} \)

Group 6—Reduction of Radicals

1. \( a^4 \sqrt{a} \)
2. \( a^5 \sqrt{a} \)
3. \( 2a^3 \sqrt{3a} \)
4. \( 3a \sqrt{a} \)
5. \( 12a \sqrt{1} \)

Group 7—Fractional Exponents

1. \( y \sqrt{y} \)
2. \( y^3 \sqrt{y} \)
3. \( 2 \sqrt{y^2} \)
4. \( 2 \sqrt{y^4} \)
5. \( 4 \sqrt{y^8} \)
Part IV—Graphs

Directions: On blank space provided solve each problem and place answers in parentheses at the right locate each point and draw the graph for equation \( y = x^2 - 2x - 3 \). One point for each point correctly found and one point for points correctly located.

<table>
<thead>
<tr>
<th>( x )</th>
<th>( y )</th>
</tr>
</thead>
<tbody>
<tr>
<td>58-59.</td>
<td>5</td>
</tr>
<tr>
<td>60-61.</td>
<td>4</td>
</tr>
<tr>
<td>62-63.</td>
<td>3</td>
</tr>
<tr>
<td>64-65.</td>
<td>2</td>
</tr>
<tr>
<td>66-67.</td>
<td>1</td>
</tr>
<tr>
<td>68-69.</td>
<td>0</td>
</tr>
<tr>
<td>70-71.</td>
<td>-1</td>
</tr>
<tr>
<td>72-73.</td>
<td>-2</td>
</tr>
<tr>
<td>74-75.</td>
<td>-3</td>
</tr>
</tbody>
</table>

Part V—Equations to Solve and Written Problems

Group 1—Equations to Solve

Directions: On blank space provided solve each problem and place answers in parentheses at right.

75-78. (4 points) \( \frac{4}{x+2} + \frac{7}{x+3} = \frac{x^2 + 44}{x^2 + 5x + 6} \)

\( x = (............) \)

or \( (............) \)
79-80. (2 points) Solve by factoring. Show work. \(3x^2 - 5x - 2 = 0\)

81-84. (4 points) Solve by completing the square. Show work. \(4x^2 - 8x + 3 = 0\)

85-89. (5 points) Solve by use of the formula. Show work. \(2x^2 - 3x + 1 = 0\)
Group 2—Written Problems

Directions: Solve problems in space provided and place answers in space provided at right of problem.

90-94. (5 points) If twice the square of a number is decreased by three times the number the result is 35. Find the numbers.

Number = (..........)

or = (..........)

95-100. (5 points) A can do a certain work in 15 days, B can do the same work in 12 days. A works for 3 days and asks B to help him finish the work. How many days will B have to work with A to complete the work?

B works (.......... days)
INSTRUCTIONS TO STUDENTS:
This examination consists of five parts. The problems are grouped according to difficulty and time required for solving. Below is a table showing value of correct answers in each part.

<table>
<thead>
<tr>
<th>Part I</th>
<th>Each problem counts 1 point (Total, 35)</th>
</tr>
</thead>
<tbody>
<tr>
<td>Part II</td>
<td>Each problem counts 2 points (Total, 50)</td>
</tr>
<tr>
<td>Part III</td>
<td>Each problem counts 3 points (Total, 30)</td>
</tr>
<tr>
<td>Part IV</td>
<td>Each problem counts 4 points (Total, 20)</td>
</tr>
<tr>
<td>Part V</td>
<td>Each problem counts 5 points (Total, 15)</td>
</tr>
</tbody>
</table>

Part I. (Each problem, 1 point)

1. Simplify \(12a - (-4a)\).
2. Subtract \(9a^2b^2\) from \(-8a^2b^2\).
3. Solve for \(x\): \(\frac{6}{5x - 3} = x\).
4. Multiply \(\left(\frac{a}{b} + 1\right)\) by \(\left(\frac{a}{b} - 1\right)\).
5. Simplify \((6xy^2 - 3xy) \div 3xy\).
6. Find two factors of \(8x - 12y\).
7. Reduce to simplest form \(\frac{1}{x^{3/2}}\).
8. Solve for \(x\): \(\frac{3x^3}{3} = 0.8\).
9. What is the perimeter of a square field with an area of \(16a^2\) sq. rods?
10. Express in dollars the sum of \(d\) dollars and \(c\) cents.
11. Find the value of \(\left(\frac{3c^3}{5ab^4}\right)^2\).
12. What distance does an automobile travel in \((t + 2)\) hours at 40 miles per hour?
13. If \(c\) parts of cement and \(s\) parts of sand are mixed, what part of the mixture is cement?
14. Solve for \(x\): \(-3x = 8 - (5 + 2x)\).
15. Multiply: \((6x - 5y)^2\).
16. Divide: \(-28r^3s\) by \(-7rs\).
17. If \( x = 3, y = -2 \), \( x^2y^2 = \) 

18. If \( a = 2, b = 3, c = -4 \), find the value of \( 2c^2 - ac + b^2 \)

19. Perform the indicated operation:
\[
\frac{14x^3}{15y^5} \div \frac{7y}{5x}
\]

20. Subtract \( 2x^3 - 3x^2 + 4x - 5 \) from 0.

21. If 10 is 4 more than 2a, what is the value of \( 3a - 5 \)?

22. Write the following statement as an equation: If \( b \) is subtracted from 7 times a certain number, the result equals 5 times the number.

23. Simplify \( \left( -\frac{1}{2} \right)^3 \cdot a^3 \)

24. Find the value of \( L \):
\[ L = a + (n-1)d, \text{ when } a = 10, d=5, n=6 \]

25. Write as an improper fraction:
\[ 3x + 2y - \frac{5}{x} \]

In the following 5 problems answer true or false.

26. The square of the sum of \( a \) and \( b \) is \( a^2 + b^2 \)

27. \( 3a^2 \) means \( 3a \cdot 3a \)

28. In the expression \( -5xy \), the coefficient of \( x \) is \(-5y\)

29. The product of two numbers with like signs will always be a number having this same sign.

30. The ratio of one number to another is found by comparing them by division.

In the next 5, fill in blanks with suitable words.

31. The terms \( 2x, 7x, -9x \) are called _____ terms.

32. In the expression \( 4a^3 \), the _____ of \( a \) is 3.

33. An algebraic expression in which the parts are not separated by the signs \( \div \) or \( - \) is called a ________.

34. Doubling the length of the radius of a circle multiplies the area by ________

35. Two angles are called supplementary if their sum is ________°.
Part II. (Each problem, 2 points)

36. Factor $2x^2 - 11x + 12$.

37. Simplify: $\frac{1 + \frac{1}{x}}{1 - \frac{1}{x}}$.

38. Divide $6x^3 - 25x^2 - 8 + 26x$ by $3x - 2$.

39. Solve for $x$: $5(4x + 3) - (9x + 5) = 7x - 30$.

40. Reduce to lowest terms: $\frac{x + y}{x^2 + 2xy + y^2}$.

41. Find the least common denominator of $\frac{3x}{x-y}$, $\frac{5x}{x^2-2xy+y^2}$.

42. Simplify: $\frac{3ab^2 - 6a}{5b^3 - 10b}$.

43. Simplify: $\frac{x}{x-4} - \frac{x}{x+4}$.

44. Factor $9a^5 - 36a^3$.

45. A tower casts a shadow of 20 feet. A man 6 ft. tall casts a shadow of $1\frac{1}{2}$ ft. Write the proportion that would be used in solving the problem.

46. Divide $8x^3 + y^3$ by $2x + y$.

47. Multiply: $(3x^2 - 7 + 4x)(2x - 3)$.

48. Solve by factoring: $x^2 = 4x$.

49. Find prime factors: $a^2 + 2ab + b^2 - x^2$.

50. The sum of two numbers is 53, and 3 times the larger added to 4 times the smaller equals 173. Find the numbers.

51. Simplify: $\frac{x^2 - 13 + x^2}{x + 1 - x}$.

52. Solve for $y$: $(y - 3)^2 - (y - 2)^2 = y - 1$.

53. A and B together have $50. If 8 times A's money is $10 more than twice B's, how many dollars has each?

54. Simplify: $\frac{24x^2 - 26x - 5}{6x^2 - 9x} \cdot \frac{2x - 3}{6x + 1}$.
55. Perform indicated division and simplify:
\[
\frac{2a^2 + 5a - 3}{2a^2 - 5a + 3} \div \frac{a^2 + 3a}{a - 1}
\]

56. What is the principal if the amount is $280 for 2 years at 6%?

57. Solve for x and y:
\[
\begin{align*}
x + 3y &= -7 \\
4x - 5y &= 23
\end{align*}
\]

59. Combine into one fraction and simplify:
\[
\frac{x - 5}{x^2 + 6x + 8} + \frac{x + 1}{3x + 12}
\]

60. Multiply and simplify:
\[
18 \left( \frac{1a}{3} - \frac{1b}{6} + \frac{1c}{2} \right) - 12 \left( \frac{1a}{4} + \frac{1b}{2} - \frac{1c}{6} \right)
\]

Part III. (Each problem, 3 points)

61. Solve for x and check:
(Solution 2 points, check 1 point)
\[
2x - (4x - 9) + 5 (x + 4) = 41
\]

Check:

62. Solve for x:
\[
\frac{x^2 - 5x}{2} = \frac{19}{5}
\]

63. Solve for V:
\[
S = Vt - \frac{1}{2}gt^2 \text{ when } S = 1328, \quad g = 32, \quad t = 4.
\]

64. Find the prime factors of \(a^8 - b^8\)

65. A can plow a field with his team in 10 days, and B can plow it in 2 days with a tractor. How many days will it take them to do it together?

66. A sum of money amounting to $2.20 consists of nickels and quarters. There are in all 16 coins, How many of each coin are there? Write equation and solve.

\[
\text{Equation: }
\]

Nickels

Quarters
67. Find prime factors of \(2x^4 - 26x^2 + 72\).

68. Missing a train by three hours, a man hires an airplane to catch it. The airplane can go 30 miles an hour faster than the train. If the man catches the train in 4 hours, what is the rate of the train in miles per hour?

69. The capacities of two motor trucks are 3 tons and 5 tons respectively. The larger truck made 6 more trips than the smaller one. How many trips did each make if they hauled a total of 88 tons?

70. A man needs 20 hr. for a trip if he goes 45 miles an hour. How fast must he go to make the trip in 18 hours? Write proportion and solve.

71. Solve by graphing:

\[
2x - y = 4
\]

\[
2x + 3y = 12
\]
72. A grocer has coffee which he sells at 36% per pound, and other coffee which he sells at 20% per pound. How many pounds of the better coffee will there be in a mixture of 100 lbs. which he may sell at 28% a pound?

73. Solve for x and y:
\[
\frac{x}{4} - \frac{y}{6} = \frac{1}{2},
\]
\[
\frac{x}{3} - \frac{y}{2} = -1.
\]

74. A man invests $6000, part at 4% and part at 7% and obtains an average of 6% on his money. How much is invested at each rate?

75. A's age now is 2/3 of B's age, but in 9 yrs. A's age will be 3/4 of B's age. Find their present ages.

76. Solve:
(See below)

77. Simplify:
\[
\frac{a}{a-b} - \frac{a}{a+b} = \frac{b}{a-b} + \frac{a}{a+b}
\]

78. A crew can row 10 miles downstream in 50 minutes, and 12 miles upstream in 1 \frac{1}{2} hrs. Find the rate in miles an hour of the current and of the crew in still water.

76. (cont'd) \[
\begin{align*}
\frac{4a + b}{5} - \frac{6a - 3b}{3} &= -3 \\
\frac{8a + 5b}{2} - \frac{10a - b}{7} &= -4
\end{align*}
\]
ADDIION AND SUBTRACTION

Carefully perform the operations as indicated.

1. \(4r + 3r + 2r =\)
2. \(2x + 3x =\)

3. \(12b + 6b - 3b =\)
4. \(2c + \frac{1}{2}c =\)

5. \(7x - x + 6 - 4 =\)
6. \(3a - 4b + 5a - 2b =\)

7. \(5m + (-4m) =\)
8. \(20x - (10x + 5x) =\)

9. \((4r - 5t) + (s - 3r) =\)
10. \(8c - (-6 + 3c) =\)

11. \(3a^2 - 3b - (2a^2 + 3b - 4) =\)
12. \(5x - [4x - (3x - 1)] =\)

13. \(\frac{3c}{4} - \frac{3c}{8} =\)
14. \(\frac{3x - 2}{3} + \frac{x + 4}{6} =\)

(Turn over. There are exercises on the other side)
15. \( \frac{1}{a-x} - \frac{3x}{a^2-x^2} = \)

16. \( \frac{r}{r+z} + \frac{r}{r-z} = \)

17. \( \frac{5a+1}{6a} - \frac{3a-2}{2a} = \)

18. \( 3 - \frac{3-2x}{4} - 2x = \)

19. \( \frac{10x+3y}{2x^2y} - \frac{3x+5y}{xy^2} = \)

20. \( \frac{1}{a+1} - \frac{a}{a^3-a+1} - \frac{a-4}{a^3+1} = \)

21. \( \frac{2}{x^2-5x+6} - \frac{15}{x^2+2x-15} = \)

22. \( \frac{3-2x}{(x-1)^3} + \frac{x+1}{(x-1)^2} - \frac{1}{(x-1)} = \)

23. \( \sqrt{20} + \sqrt{45} + \sqrt{\frac{1}{5}} = \)

24. \( \frac{a}{a-2} - \frac{a-2}{a+2} + \frac{3}{4-a^2} = \)
MULTIPLICATION AND DIVISION

Carefully perform the operations as indicated. Reduce all answers to their simplest forms.

1. \(3 \cdot 7y = \)

2. \(\frac{12n}{4} = \)

3. \(2a \cdot 4ab^2 = \)

4. \(6c^3 \div 2c^2 = \)

5. \(\frac{3}{4} \text{ of } 9m = \)

6. \(\frac{-8a^2b}{4a^2} = \)

7. \(4x \cdot (-3xy^3) = \)

8. \(a^3 \cdot (-3a) \cdot (-2a) = \)

9. \(\frac{18m^2n - 27mn^2}{9mn} = \)

10. \(\frac{4x^4}{5} \div 2x^2 = \)

11. \((2a^2 + 7a - 9)(5a - 1) = \)

12. \(\frac{n^4 + 7n^2 - 30}{n^2 - 3} = \)

13. \(\frac{7a}{15} + \frac{7a^2}{20} = \)

14. \(\frac{-12x^2y^2 \cdot (x - 2)}{-3x^2y^2} = \)

(Turn over. There are exercises on the other side)
15. \( \frac{m + n}{a} \cdot \frac{b}{m^2 - n^2} = \)

16. \((-3xy^3)^4 = \)

17. \(\frac{c^4 - d^4}{(c - d)^2} \cdot \frac{c - d}{c^2 + d^2} = \)

18. \(3x^{\frac{1}{2}} \cdot 4x^{\frac{3}{2}} = \)

19. \(\frac{a^2 + \frac{3}{2}a - 1}{a + 2} = \)

20. \(\frac{p^2 + 4p - 45}{p^2 + 2p + 4} \cdot \frac{p^3 - 8}{p^2 - 81} \cdot \frac{1}{3p^2 - 15r} = \)

21. \(\frac{x^3 + 27}{x^2 + x - 12} + \frac{3x + 9}{x + 4} = \)

22. \(64^\frac{3}{2} \times 27^\frac{1}{3} = \)

23. \(\frac{3 \sqrt{6a}}{2a \sqrt{18}} \cdot \sqrt{12a} = \)
Solve the following equations and formulae:

1. \(2x = 4\).
2. \(7m = 3m + 12\).

3. \(3x + 3 = 9\).
4. \(5a + 5 = 61 - 3a\).

5. \(7n - 12 - 3n + 4 = 0\).
6. \(10 - 11z = 4 - 8z\).

7. \(\frac{2}{3}z = 6\).
8. \(c - 2(3 - 4c) = 12\).

9. \(\frac{1}{2}x + \frac{1}{4}x = 3\).
10. \(\frac{2x}{3} = \frac{5}{8}\).

11. The area of a triangle = \(\frac{1}{2}bh\), in which
    \(b = \) length of the base
    and \(h = \) height of the triangle.

   How many square feet are there in the area of a triangle whose base is 10 feet, and whose height is 8 feet?

12. \(\frac{y}{3} = \frac{5}{2} - \frac{y}{4}\).
13. \(\frac{1}{4}(x + 5) = 5\).

(Turn over. There are exercises on the other side)
14. \[3m + 7n = 34\] 
\[7m + 8n = 46\]

15. \[\frac{4}{3 - x} = \frac{2}{1 + x}\]

16. The area of a circle = \(\pi r^2\), in which 
\(\pi = \text{radius of the circle}\) 
and \(\pi = 3\frac{1}{7}\).

Find the area in square feet of circle whose radius is 7 feet.

17. In the formula \(RM = El\), find value of \(M\).

18. \[\frac{x + 3}{x - 2} = \frac{x + 5}{x - 4}\]

19. \[p^2 - 5p = 50\]

20. \[\frac{2}{x^2 + 4x + 3} = \frac{3}{x^2 + 3x + 2}\]

21. \[\frac{1}{x} + \frac{2}{y} = 1\]

\[\frac{4}{x} - \frac{4}{y} = 1\]

22. \(F\) = temperature in Fahrenheit degrees 
\(C\) = temperature in Centigrade degrees 
and \(F = \frac{9C}{5} + 32^\circ\).

Solve for \(C\) when \(F = 70^\circ\).

23. \[\frac{6x - 2}{x + 3} - 3 = \frac{3x^2 + 13}{x^2 - 9}\]

24. \(S = \frac{1}{2}gt^2\); solve for \(t\).

25. \[\sqrt{x^2 - 1} - x = -1\]
FIRST YEAR ALGEBRA SCALES

By HENRY G. HOTZ

Published by Teachers College, Columbia University. Copyright, 1920, by Teachers College

PROBLEMS

Do not work out the answer to the problem—merely indicate the answer or state the equation in each case.

1. If one coat cost $x$ dollars, how much will 3 coats cost?
   Answer.

2. A man is $m$ years old; how old was he $r$ years ago?
   Answer.

3. A boy has $a$ marbles and buys $b$ more; how many has he then?
   Answer.

4. A gold watch is worth ten times as much as a silver watch, and both together are worth $132$. How much is each worth?
   Equation.

5. The distance from Chicago to New York by rail is 980 miles. If a train runs $v$ miles an hour, what is the time required for the run?
   Answer.

6. The width of a basket ball court is 20 feet less than its length. The perimeter of the court (distance around) is 240 feet. Find the dimensions.
   Equation.

7. The total number of circus tickets sold was 836. The number of tickets sold to adults was 136 less than twice the number of children's tickets. How many were sold of each?
   Equation.

   (Turn over. There are problems on the other side)
8. A rectangular box is $d$ inches deep, $w$ inches wide, and contains $r$ cubic inches. What is its length?

Answer.

9. The area of a square is equal to that of a rectangle. The base of the rectangle is 12 feet longer and its altitude 4 feet shorter than the side of the square. Find the dimensions of both figures.

Equation.

10. A tower casts a shadow of 20 feet. A man, 5 feet 9 inches high, who is near at the same time, casts a shadow of 2 feet 6 inches. Find the height of the tower.

Proportion.

11. Five thousand dollars is invested in two banks, part in one at 3 per cent and the rest in the other at 4 per cent. The annual income from the two investments is $172. How much is each investment?

Equation.

12. A train leaves a station and travels at the rate of 40 miles an hour. Two hours later a second train leaves the same station and travels in the same direction at the rate of 55 miles an hour. Where will the second train pass the first?

Equation.

13. A merchant has two kinds of tea, one kind costing 50 cents and the other 65 cents per pound. How many pounds of each must be mixed together to produce a mixture of 20 pounds that shall cost 60 cents per pound?

Equation.

14. An open box is made from a square piece of tin by cutting out a 5-inch square from each corner and turning up the sides. How large is the original square, if the box contains 180 cubic inches?

Equation.