A MEASURE OF THE VALUE OF RECORDINGS
IN TEACHING ARITHMETIC

by

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<td>45</td>
</tr>
</tbody>
</table>
CHAPTER I

THE PROBLEM AND DEFINITIONS OF TERMS USED

I. STATEMENT OF THE PROBLEM

It was the purpose of this experiment to determine the effectiveness of teaching certain processes in elementary arithmetic through the use of instructional recordings as compared with instruction through customary instructional procedures.

II. IMPORTANCE OF THE STUDY

The field of audio-visual education has not been thoroughly explored. Much advancement has been made in the use of visual aids in education. Auditory aids have not been applied to the field of education as have visual aids. Recordings have been used widely in the fields of music and speech improvement. Some use has been made of them in providing directions for standardized tests. McKown and Roberts report their use in "connection with all or nearly all school subjects."1 Also, certain companies are now presenting current events and historical dramas by this means.

means. Lectures on technical subjects have been presented in this way. The use of the recording in this experiment differs from their previous uses in two ways. First, the material presented is highly factual and is used entirely for purposes of instruction in connection with new materials. Second, the recordings were supplemented by a study sheet designed to accompany the materials presented.

Recordings, as one type of auditory aid, may claim four advantages. First, instructional material which has been recorded may be repeated with minimum effort at a desired time. Second, this repetition through recordings tends to clarify misunderstanding more easily than the regular verbal method of repetition because the vocabulary, voice inflection, sentence structure, enunciation, and voice quality remain the same. Third, when material is presented by recordings the learner is placed in direct contact with the learning situation. There is no teacher interference to create embarrassment or confusion. Fourth, materials prepared for use on recordings are likely to be of higher quality than materials presented more extemporaneously.

The customarily accepted advantages of the conventional method of teaching are probably more familiar to both pupil and teacher. First, extended explanation and discussion may become an integral part of the presentation. Second, the
teacher can more successfully adjust the time factor. Third, adjustments in materials and methods may be made after the presentation has begun. Fourth, the conventional method of teaching may favor re-statement of materials for purposes of adaptation to various levels of ability. Fifth, the conventional method of teaching is more personalized.

This study should indicate the value of instructional recordings in terms of pupil achievement as measured in this particular experiment. In-as-much as this is true, this experiment may be said to be a test of the claimed advantages of both methods.

III. LIMITATIONS OF THE STUDY

This study is not based upon as many cases as was originally planned. The data from one school had to be discarded because the failure of the instructor to follow directions may have influenced the reliability of those particular data.

The groups used in this study were not entirely homogeneous. The heterogenous nature of the groups made the conditions more natural, however. In the treatment of the data gained from these two groups, the statistical significance of the difference of the means of the two groups was calculated in order to establish their value.
The presentation of prescribed materials, a necessity in this experiment, involves certain mechanistic limitations. Since there is little opportunity for digression, individual needs are more difficult to treat. Extended explanations which often lead to a more complete understanding on the part of the pupil are limited to the writer's ability to foresee those needed explanations. Although it would be possible to interpose discussion between the recordings and the progress tests, such was not the case in this experiment. Naturally then, this experiment measures understanding to the degree that efficiency in computation on the tests indicates understanding. With the exception of post-presentation discussions, these limitations apply to both methods, however.

IV. DEFINITIONS OF TERMS USED

Experimental group. Experimental group refers to that group taught through the use of recordings.

Control group. Control group refers to that group taught in the customary manner.

Recording. A recording is a term referring to a disc used to reproduce the voice sounds for purposes of instruction.

Study sheet. The study sheet was a list of examples
designed to accompany the materials presented through the use of the recording.

Survey test. The survey test was the preliminary test given to determine whether any individual in the group was well enough acquainted with the process being presented to affect the reliability of the data collected from subsequent experimentation.

Progress test. The progress test was the follow-up test given after each presentation.

Arithmetic process. The term, arithmetic process, was used to define the particular material in arithmetic presented at one time, e.g., cancellation of fractions.

V. METHOD OF PROCEDURE

The experimental method of research was used. Four arithmetic processes were used in this experiment. The control and experimental groups were established for the presentation of the first process. For each following presentation the groups were re-established, the control group being established as the experimental group and the experimental group being established as the control group. The classification of each group was determined by the method used in presenting the material.
VI. ORGANIZATION OF REMAINDER OF THESIS

This thesis is organized into four chapters. Chapter II is a discussion of the procedure involved.

In Chapter III the data are presented and analyzed. Explanations accompany the tables in this chapter.

The findings and recommendations of this study are presented in Chapter IV.

The Appendix contains the script for the recordings, the instructional materials available for the teacher, and the complete series of tests used in the four arithmetic processes.
CHAPTER II

THE PROCEDURE

It is the purpose of this chapter to explain the organization and execution of the experiment. This includes:

1. preliminary testing and group organization;
2. presentation of materials;
3. testing;
4. second presentation;
5. second testing.

The data for this study were collected from the sixth grades of the Wm. Rea School and Laboratory School of Terre Haute, Indiana, and Glenn School of Vigo County, Indiana. A fourth school, Maryland School, of Vigo County participated in a part of the experiment.

I. PRELIMINARY TESTING AND ORGANIZATION OF GROUPS

All pupils taking part in the experiment were first given the Otis Group Intelligence Scale, Advanced Examination: Form A. They were next given the Stanford Achievement Test, Intermediate Arithmetic Test: Form D. The results of these two tests were used to determine the degree of equality existing among the groups. This procedure made it possible to compare the groups in regard to both intelligence and achievement. This also made it possible to use these same data in establishing the relationship between intelligence
and achievement and between grade placement and achievement as indicated by measurements following each method.

Four arithmetic processes were taught. In teaching the first process the Wm. Rea School and the Laboratory School were established as the experimental group. Glenn School was used as the control group. In teaching the second process Glenn and Maryland Schools were used as the experimental group. The Wm. Rea School and the Laboratory School were used as the control group. In teaching the third process Glenn School and the Wm. Rea School were established as the control group while the Laboratory School was used as the experimental group. The teaching of the fourth process involved a reversal of the groups as set up in the teaching of the third process. The Laboratory School was used as the control group while the Wm. Rea School and Glenn School were used as the experimental group. This organization is illustrated in Table I. Each school with the exception of Maryland used each method two times.

II. PRESENTATION OF MATERIALS

The four arithmetic processes presented were cancellation of fractions, division of mixed numbers, multiplication of decimals, and division of decimals.

The typical procedure in presenting any one of these
TABLE I

CLASSIFICATION OF PARTICIPATING SCHOOLS

<table>
<thead>
<tr>
<th>Parts</th>
<th>Control Group</th>
<th>Experimental Group</th>
</tr>
</thead>
<tbody>
<tr>
<td>Part I</td>
<td>Glenn</td>
<td>Laboratory--Rea</td>
</tr>
<tr>
<td>Part II</td>
<td>Laboratory--Rea</td>
<td>Glenn--Maryland</td>
</tr>
<tr>
<td>Part III</td>
<td>Glenn--Rea</td>
<td>Laboratory</td>
</tr>
<tr>
<td>Part IV</td>
<td>Laboratory</td>
<td>Glenn--Rea</td>
</tr>
</tbody>
</table>

processes was as follows:

First, a survey test was given. This test contained five examples to be solved. Ample time (five minutes) was allowed for the group to complete the survey test. The test contained examples similar to those given on progress tests and in the instructional materials for that particular process. Directions for the test were given at the top of the page. Each survey test, as each progress test, was constructed by the writer, since no satisfactory standardized tests for the experiment could be found. In case an individual taking part in the experiment made a score of two or more correct it was assumed that he or she was already familiar with the material being presented. A score of one was considered a "chance" score. Subsequent data collected from those pupils making a score of two or more were discarded.
although they took part in the rest of the particular presentation in which their ineligibility was involved.

Following the survey test one of two methods was used. Either the recording was played before the group or the regular classroom teacher presented the materials in the customary verbal manner. Recordings were generally twelve minutes long while the time consumed by customary verbal presentation varied. It was, however, generally about the same as that required for the use of a recording.

It was the plan of this experiment to match the recordings against the more conventional classroom procedures. In an effort to see that the recordings stood entirely on their own merit, the pupils in the experimental groups weren't allowed to ask questions. All information which they gained for use on the progress tests came from the recordings. The control group, on the other hand, asked questions as in usual classroom procedure.

In those cases involving the use of the recording a study sheet was given to each pupil. Each pupil followed the example on his study sheet as the example was being talked about on the recording. The study sheet was made up of examples to be discussed on the recording. These examples were already solved. The instructions on the recording undertook to explain the method by which these solutions were
obtained. In addition to the examples directly related to the verbal material on the recording there were examples for the pupil to solve. By the figure designating the number of these examples there was often another figure referring the pupil back to a similar example, the solution of which was prepared for him and which had been previously explained through the use of the recording. The use of the study sheet in connection with a recording is an important innovation.

The customary verbal presentation was likewise preceded by the same survey test as preceded the recorded materials in the presentation of the respective processes. However, in an effort to more nearly simulate the normal classroom procedure with the control group the examples were placed on the chalkboard instead of being presented to the pupil through the use of study sheets. The examples presented on the chalkboard to the control group were the same as were presented to the experimental group through the use of study sheets. The teacher then presented essentially the same material in the customary verbal manner that had been presented to the experimental group through the use of the recordings. This material was adapted for the control group from the script of the recorded material. The teacher was free to modify the material prepared for her use in order to keep within the normal range of her methods. Usually the teacher pointed to
the examples on the board as these examples were discussed. The pupils were given a sheet of paper on which to work the unsolved examples on the chalkboard. Questions were often asked by the pupils, whereas, as has been pointed out earlier, those pupils in the experimental groups were not allowed to ask questions. The number of questions asked varied with the arithmetic processes and with the class.

III. TESTING

Following the presentation of materials, a test was given. This test was given to both the experimental groups and the control groups upon the completion of the presentation of each arithmetic process. This test was known as Progress Test I in each of the processes. Thus, upon completing the presentation of the materials relating to cancellation of fractions, Progress Test I in cancellation of fractions was given. A similar statement could be made about each of the other arithmetic processes. The progress test was constructed so as to follow the information presented through the use of the recording or through the customary teaching procedure. The progress test usually contained fifteen examples. It was always given with a ten-minute time limit. Time limits were the same for both the control and experimental groups.
IV. SECOND PRESENTATION

In order to discover some indication of the value of a single repetition the materials under each process were presented a second time. This was true for both the control group and for the experimental group. Thus, the presentation of any one arithmetic process such as multiplication of decimals, Progress Test I was followed by a second playing of the recording or by repetition of the verbal instruction by the teacher. This latter presentation was, of course, as nearly a duplicate of the first presentation as possible.

V. SECOND TESTING

The second presentation of the materials in each process was followed by another test known as Progress Test II. It was constructed in the same way and for the same purposes as Progress Test I. The examples of Progress Test II never duplicated the examples of Progress Test I. The ten-minute time limit was used with Progress Test II as with Progress Test I. All other factors were as nearly the same as possible.

Briefly, then, the presentation of any one arithmetic process consisted of: (1) a survey test; (2) presentation of materials of instruction through the use of the recordings or
through the customary verbal procedure; (3) the first progress test; (4) a second presentation of materials of instruction; and (5) a second progress test.
CHAPTER III

PRESENTATION AND ANALYSIS OF DATA

The results of this experiment are to be discussed in this chapter. The discussion includes (1) an analysis of the participating groups from the standpoints of general intelligence and achievement in arithmetic, (2) a comparison of the value of the two methods as indicated by the mean scores of Progress Tests I and II in each presentation, and (3) a comparison of the coefficients of correlation between intelligence and achievement on the progress tests and grade placement and achievement on the progress tests in the control groups with coefficients of correlation between achievement on the progress tests and the same two factors in the experimental groups.

FINDINGS FROM THE INTELLIGENCE TESTS

The Otis Group Intelligence Scale, Advanced Examination: Form A was first given to the six-B pupils in each participating school.

In Part I (the first presentation) of the experiment Glenn School with twenty-two pupils was established as the control group while the Laboratory and Rea Schools with a combined total of forty-five pupils were established as the...
experimental group. More equitable numerical groups were
planned but the data from the school assigned to work with
Glenn School in the control group had to be discarded for
reasons already stated. Table II shows that the mean I. Q.
of the control group was 101, with a range from an I. Q. of
83 to an I. Q. of 118.

The mean I. Q. of the experimental group in Part I was
104.5, with a range from an I. Q. of 77 to an I. Q. of 122.
This is shown in Table III, pages 18 and 19. It will be
noted that the control group had a shorter range of I. Q.
scores but the experimental group had a higher mean score.
The difference between the mean intelligence quotients of
the control and experimental groups in this part is greater
than in any other part of the experiment. As will be seen
later, the chances in one hundred that the difference between
the true means is more than zero are greater in this part
than in any other part of the experiment dealing with I. Q.
scores. This tends to establish the superiority of the ex­
perimental group in this item.

In Part II, division of mixed numbers, the Laboratory
and Rea Schools with thirty-three pupils were established as
the control group. Table IV, page 20, indicates that the
mean I. Q. of the control group was 103.5 with a range from
an I. Q. of 77 to an I. Q. of 122.
TABLE II
CONTROL GROUP (GLENN SCHOOL) RANKED I. Q., GRADE PLACEMENT, AND SCORES ON PROGRESS TESTS I AND II, PART I

<table>
<thead>
<tr>
<th>Pupil Number</th>
<th>I. Q.</th>
<th>Grade Placement</th>
<th>Progress Test I</th>
<th>Progress Test II</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>118</td>
<td>7.2</td>
<td>16</td>
<td>16</td>
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<tr>
<td>2</td>
<td>117</td>
<td>5.1</td>
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<tr>
<td>3</td>
<td>114</td>
<td>6.4</td>
<td>16</td>
<td>14</td>
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<tr>
<td>4</td>
<td>110</td>
<td>6.3</td>
<td>16</td>
<td>14</td>
</tr>
<tr>
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<td>5.5</td>
<td>11.8</td>
<td>11.7</td>
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</tbody>
</table>

The following facts may be found in Table V, pages 21 and 22. The mean I. Q. of the experimental group in Part II was 101, with a range from an I. Q. of 83 to an I. Q. of 117. The difference between the mean intelligence quotients of the control and experimental groups was somewhat less than the same factor in Part I.
TABLE III

EXPERIMENTAL GROUP (LABORATORY AND REA SCHOOLS)
RANKED I. Q., GRADE PLACEMENT, AND SCORES ON PROGRESS TESTS I AND II, PART I

<table>
<thead>
<tr>
<th>Pupil Number</th>
<th>I. Q.</th>
<th>Grade Placement</th>
<th>Progress Test I</th>
<th>Progress Test II</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>122</td>
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<td>14</td>
<td>15</td>
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<td>2</td>
<td>121</td>
<td>6.2</td>
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<tr>
<td>3</td>
<td>121</td>
<td>6.0</td>
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TABLE III (continued)

EXPERIMENTAL GROUP (LABORATORY AND REA SCHOOLS)
RANKED I. Q., GRADE PLACEMENT, AND SCORES
ON PROGRESS TESTS I AND II, PART I

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It should be noted here that the teachers at Rea School were changed between Parts II and III. This was a matter of school administration and had no apparent effects on the experiment.

In Part III, multiplication of decimals, the Glenn and Rea Schools with thirty-nine pupils were established as the control group while the Laboratory School with thirty-two pupils was established as the experimental group. The mean I. Q. of the control group was 104.3, with a range from an I. Q. of 83 to an I. Q. of 127. This is shown in Table VI, page 23.

As shown by Table VII, page 24, the mean I. Q. of the experimental group in Part III was 106.4, with a range from an I. Q. of 77 to an I. Q. of 122. The difference between
TABLE IV

CONTROL GROUP (LABORATORY AND REA SCHOOLS) RANKED I. Q., GRADE PLACEMENT, AND SCORES ON PROGRESS TESTS I AND II, PART II

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TABLE V (continued)

EXPERIMENTAL GROUP (GLENN AND MARYLAND SCHOOLS).
RANKED I. Q., GRADE PLACEMENT, AND SCORES
ON PROGRESS TESTS I AND II, PART II

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the mean intelligence quotients of the control and experimental group in this presentation was less than the same factor in Part II or Part I.

In Part IV, division of decimals, the Laboratory School was established as the control group, while Glenn and Rea Schools were established as the experimental groups. In this part of the experiment the control group was made up of thirty-three pupils while the experimental group was made up of forty-one pupils. By re-establishing the groups each time the numerical superiority never remained with either the control or experimental group. Any significance that may be attached to this factor is somewhat cancelled by the calculating of the statistical significance of the difference of the means.

In this part of the experiment the control group had
TABLE VI

CONTROL GROUP (GLENN AND PEA SCHOOLS) RANKED I. Q., GRADE PLACEMENT, AND SCORES ON PROGRESS TESTS I AND II, PART II

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Mean 104.3 5.6 12.9 13.8
### Table VII

**Experimental Group (Laboratory School) Ranked I. Q., Grade Placement, and Scores on Progress Tests I and II, Part II**

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a mean I. Q. of 105.6, with a range from an I. Q. of 77 to an I. Q. of 122. This is shown in Table VIII. The mean I. Q. of the experimental group was 103.7, with a range from an I. Q. of 83 to an I. Q. of 127. This is shown in Table IX, pages 27 and 28. The difference between the mean intelligence quotients of the control and experimental groups in this part was approximately the same as the identical factor in Part III.

FINDINGS FROM THE ARITHMETIC ACHIEVEMENT TESTS

After intelligence testing was completed the Stanford Achievement Test, Intermediate Arithmetic Test: Form D was given. The groups were the same as those indicated in the discussion on intelligence testing. Since this is true the number of pupils involved would be exactly the same as would also be the names of the participating schools.

Table II, page 17, shows that in Part I, cancellation of fractions, the mean grade placement for the control group was 5.5, with a range from a grade placement of 3.6 to 7.2. The experimental group had a grade placement mean of 5.3 with a range from a grade placement of 3.8 to 6.0. This is shown in Table III, pages 18 and 19. It will be noted that the control group had a greater grade placement score. Likewise, they had a greater range of scores. The difference between the mean scores of the control and experimental groups was two months. This is not a great difference.
TABLE VIII

CONTROL GROUP (LABORATORY SCHOOL) RANKED I. Q., GRADE PLACEMENT, AND SCORES ON PROGRESS TESTS I AND II, PART IV

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## TABLE IX

EXPERIMENTAL GROUP (GLENN AND REA SCHOOLS) RANKED
I. Q., GRADE PLACEMENT, AND SCORES ON
PROGRESS TESTS I AND II, PART IV

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In Part II, division of mixed numbers, the mean grade placement for the control group was 5.1, with a range from a grade placement of 3.8 to a grade placement of 6.0. This is shown in Table IV, page 20. The experimental group had a grade placement mean of 5.3 with a range from a grade placement of 3.6 to a grade placement of 6.2. This is shown in Table V, pages 21 and 22. A difference of two months between the means of the control and experimental groups may be noted here. This is the same difference as existed between the control and experimental groups in Part I. In no case does the difference in mean grade placement scores of the control and experimental groups differ more than two months. This will be seen in the discussion of Parts III and IV.

In Part III, multiplication of decimals, the mean grade placement for the control group was 5.6, with a range from a grade placement of 3.6 to a grade placement of 6.6. The
experimental group had a mean grade placement of 5.4, with a range from a grade placement of 3.8 to a grade placement of 6.0. This is shown in Table VII, page 24. A difference of two months may be noted here as in Parts I and II.

In Part IV, division of decimals, the mean grade placement for the control group was 5.4, with a range from a grade placement of 3.8 to a grade placement of 6.0. This is given in Table VIII, page 26. The experimental group had a mean grade placement of 5.5 with a range from a grade placement of 3.6 to a grade placement of 6.6. In this part the difference between the mean grade placement of the control and experimental groups is less than in any of the three previous presentations.

FINDINGS FROM PROGRESS TEST I

Progress Test I was given immediately after the presentation of instructional materials relating to each part of the experiment. This test was given in the same form to both the control and experimental groups.

The following facts are taken from Table II, page 17 and Table III, pages 18 and 19. In Part I, cancellation of fractions, the mean score for the control group was 11.8. The mean score for the experimental group was 7.7. In this test as in nearly all of the progress tests the range generally ran from zero to a perfect score. Since this is
true, the subsequent discussion of progress test scores will not include a discussion of the range of scores. A discussion of the differences between the mean scores of the control and experimental groups will be included later in this report.

In Part II, division of mixed numbers, the mean score on Progress Test I for the control group was 8.5. The mean score for the experimental group was 6.1. These facts are given in Table IV, page 20 and Table V, pages 21 and 22.

In Part III, multiplication of decimals the mean score of the control group, Progress Test I, was 12.9. The mean score of the experimental group was 10.7. See Table VI, page 23 and Table VII, page 24.

In Part IV, division of decimals, the mean score of the control group, Progress Test I, was 8.5. The mean score of the experimental group on the same test was 10.3. See Table VIII, page 26 and Table IX, pages 27 and 28.

FINDINGS FROM PROGRESS TEST II

Progress Test II was given immediately after the second presentation of the instructional materials relating to each part of the experiment. This test, as Progress Test I, was given in the same form to both the control and experimental groups.

In Part I, cancellation of fractions, the mean score for the control group, Progress Test II, was 11.7. The mean
score for the experimental group was 9.6. These facts are given in Table II, page 17 and Table III, pages 18 and 19.

In Part II, division of mixed numbers, the mean score of the control group was 9.0. The mean score for the experimental group was 6.4. See Table IV, page 20 and Table V, pages 21 and 22.

In Part III, multiplication of decimals, Progress Test II, the mean score for the control group was 13.8. The mean score for the experimental group was 13.3. These facts are given in Table VI, page 23 and Table VII, page 24.

In Part IV, division of decimals the mean score for the control group was 10.2. The mean score for the experimental group was 11.1. See Table VIII, page 26 and Table IX, pages 27 and 28.

In observing the various scores made on the different progress tests by the different groups it should be noted that these mean scores are less meaningful unless two important factors affecting their value are considered. The first factor is the relationship of these scores to each other when their value is revealed through coefficients of correlation between achievement on the progress tests and abilities. The second factor is the relationship existing between these various mean scores and the intelligence quotients and achievement ratings of the individuals taking the tests.
Tables X, XI, XII, and XIII, pages 33, 34, 35, and 36, respectively attempt to fix the value of the mean scores of the progress tests in light of these two factors.

Table X deals with the progress of the two groups on materials dealing with cancellation of fractions. There is a difference of mean I. Q. scores of 3.5. There are 88 chances in 100 that the difference between the true means is greater than zero. The achievement in arithmetic as indicated by grade placement scores indicates a difference of two months. (Two-tenths is regarded as two months by the authors of the test). However, there are only 57 chances in 100 that the difference between true means is greater than zero. A point bolstering the given difference is the fact that the same groups, matched in the same way, later show the same difference with much greater statistical significance attached to it.

Continuing with Table X it may be seen that data on Progress Test I, show, with complete statistical significance, that the control group made the superior score. The mean score of the control group was 11.8 while that of the experimental group was 7.7. Data on Progress Test II shows with high statistical significance that the margin of advantage as indicated by the difference of means was somewhat reduced. On Progress Test I the difference of the means was 4.1 while on Progress Test II the same item was 2.1.
### TABLE X

**DIFFERENCES OF MEANS OF I.Q., GRADE PLACEMENT, AND PROGRESS TESTS SCORES, PART I**

<table>
<thead>
<tr>
<th>Schools</th>
<th>Classification</th>
<th>Test</th>
<th>Mean</th>
<th>Difference of means in favor of Glenn School</th>
<th>Probable error of difference of means</th>
<th>Critical ratio</th>
<th>Chances in 100</th>
</tr>
</thead>
<tbody>
<tr>
<td>Glenn Control</td>
<td>I. Q.</td>
<td>101</td>
<td>-3.5</td>
<td>1.992</td>
<td>1.76</td>
<td></td>
<td>88</td>
</tr>
<tr>
<td>Rea-Laboratory</td>
<td>Experimental</td>
<td>I. Q. 104.5</td>
<td>-3.5</td>
<td>1.992</td>
<td>1.76</td>
<td></td>
<td>88</td>
</tr>
<tr>
<td>Glenn Control</td>
<td>G. P.*</td>
<td>5.5</td>
<td>0.2</td>
<td>0.838</td>
<td>0.24</td>
<td></td>
<td>57</td>
</tr>
<tr>
<td>Rea-Laboratory</td>
<td>Experimental</td>
<td>G. P. 5.3</td>
<td>0.2</td>
<td>0.838</td>
<td>0.24</td>
<td></td>
<td>57</td>
</tr>
<tr>
<td>Glenn Control</td>
<td>P. T.I**</td>
<td>11.8</td>
<td>4.1</td>
<td>0.886</td>
<td>4.63</td>
<td></td>
<td>100</td>
</tr>
<tr>
<td>Rea-Laboratory</td>
<td>Experimental</td>
<td>P. T.I 7.7</td>
<td>4.1</td>
<td>0.886</td>
<td>4.63</td>
<td></td>
<td>100</td>
</tr>
<tr>
<td>Glenn Control</td>
<td>P. T.II</td>
<td>11.7</td>
<td>2.1</td>
<td>0.823</td>
<td>2.55</td>
<td></td>
<td>96</td>
</tr>
<tr>
<td>Rea-Laboratory</td>
<td>Experimental</td>
<td>P. T.II 9.6</td>
<td>2.1</td>
<td>0.823</td>
<td>2.55</td>
<td></td>
<td>96</td>
</tr>
</tbody>
</table>

*In this table, as in all other tables in this study, G. P. means Grade Placement.*

**In this table, as in all other tables in this study, P. T. means Progress Test.*
<table>
<thead>
<tr>
<th>Schools</th>
<th>Classification</th>
<th>Test</th>
<th>Mean</th>
<th>Difference of means in favor of Rea-Laboratory</th>
<th>Probable error of difference of means</th>
<th>Critical ratio in 100</th>
</tr>
</thead>
<tbody>
<tr>
<td>Glenn-Maryland</td>
<td>Experimental</td>
<td>I. Q.</td>
<td>101</td>
<td>2.5</td>
<td>1.600</td>
<td>1.62</td>
</tr>
<tr>
<td>Rea-Laboratory</td>
<td>Control</td>
<td>I. Q.</td>
<td>103.5</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Glenn-Maryland</td>
<td>Experimental</td>
<td>G. P.</td>
<td>5.3</td>
<td>-.2</td>
<td>.885</td>
<td>.23</td>
</tr>
<tr>
<td>Rea-Laboratory</td>
<td>Control</td>
<td>G. P.</td>
<td>5.1</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Glenn-Maryland</td>
<td>Experimental</td>
<td>P. T.I</td>
<td>6.2</td>
<td>2.3</td>
<td>.699</td>
<td>3.29</td>
</tr>
<tr>
<td>Rea-Laboratory</td>
<td>Control</td>
<td>P. T.I</td>
<td>8.5</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Glenn-Maryland</td>
<td>Experimental</td>
<td>P. T.II</td>
<td>6.4</td>
<td>2.6</td>
<td>.692</td>
<td>3.76</td>
</tr>
<tr>
<td>Rea-Laboratory</td>
<td>Control</td>
<td>P. T.II</td>
<td>9.0</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Schools</td>
<td>Classification</td>
<td>Test</td>
<td>Mean</td>
<td>Difference of means in favor of Laboratory</td>
<td>Probable error of difference of means</td>
<td>Critical ratio</td>
</tr>
<tr>
<td>---------------</td>
<td>----------------</td>
<td>------</td>
<td>------</td>
<td>--------------------------------------------</td>
<td>---------------------------------------</td>
<td>----------------</td>
</tr>
<tr>
<td>Laboratory</td>
<td>Experimental</td>
<td>I.Q.</td>
<td>106.4</td>
<td>2.1</td>
<td>1.734</td>
<td>1.203</td>
</tr>
<tr>
<td>Glenn-</td>
<td>Control</td>
<td>I.Q.</td>
<td>104.3</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Rea</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Laboratory</td>
<td>Experimental</td>
<td>G.P.</td>
<td>5.4</td>
<td>-.2</td>
<td>.40</td>
<td>1.493</td>
</tr>
<tr>
<td>Glenn-</td>
<td>Control</td>
<td>G.P.</td>
<td>5.6</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Rea</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Laboratory</td>
<td>Experimental</td>
<td>P.T.I</td>
<td>10.7</td>
<td>-2.2</td>
<td>.582</td>
<td>3.771</td>
</tr>
<tr>
<td>Glenn-</td>
<td>Control</td>
<td>P.T.I</td>
<td>12.9</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Rea</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Laboratory</td>
<td>Experimental</td>
<td>P.T.I</td>
<td>13.3</td>
<td>-.5</td>
<td>.377</td>
<td>1.271</td>
</tr>
<tr>
<td>Glenn-</td>
<td>Control</td>
<td>P.T.I</td>
<td>13.8</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Rea</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
</tbody>
</table>
### TABLE XIII
DIFFERENCES OF MEANS OF I. Q., GRADE PLACEMENT, AND PROGRESS TEST SCORES, PART IV

<table>
<thead>
<tr>
<th>Schools</th>
<th>Classification</th>
<th>Test</th>
<th>Mean</th>
<th>Difference of means in favor of Laboratory</th>
<th>Probable error of difference of means</th>
<th>Critical ratio</th>
<th>Chances in 100</th>
</tr>
</thead>
<tbody>
<tr>
<td>Laboratory</td>
<td>Control</td>
<td>I. Q.</td>
<td>105.6</td>
<td>2.0</td>
<td>1.698</td>
<td>1.236</td>
<td>80</td>
</tr>
<tr>
<td>Glenn-Rea</td>
<td>Experimental</td>
<td>I. Q.</td>
<td>103.6</td>
<td>-1.8</td>
<td>1.158</td>
<td></td>
<td></td>
</tr>
<tr>
<td>Laboratory</td>
<td>Control</td>
<td>G. P.</td>
<td>5.4</td>
<td>-0.1</td>
<td>1.133</td>
<td></td>
<td></td>
</tr>
<tr>
<td>Glenn-Rea</td>
<td>Experimental</td>
<td>G. P.</td>
<td>5.5</td>
<td>-0.1</td>
<td>0.133</td>
<td></td>
<td></td>
</tr>
<tr>
<td>Laboratory</td>
<td>Control</td>
<td>P. T.I</td>
<td>8.5</td>
<td>-1.8</td>
<td>0.705</td>
<td>2.479</td>
<td>84</td>
</tr>
<tr>
<td>Glenn-Rea</td>
<td>Experimental</td>
<td>P. T.I</td>
<td>10.3</td>
<td>-1.8</td>
<td>0.705</td>
<td></td>
<td></td>
</tr>
<tr>
<td>Laboratory</td>
<td>Control</td>
<td>P. T.II</td>
<td>10.2</td>
<td>-0.9</td>
<td>0.618</td>
<td>1.570</td>
<td>85</td>
</tr>
<tr>
<td>Glenn-Rea</td>
<td>Experimental</td>
<td>P. T.II</td>
<td>11.1</td>
<td>-0.9</td>
<td>0.618</td>
<td></td>
<td></td>
</tr>
</tbody>
</table>
In this particular presentation, then, the control group following the conventional teaching procedures proved superior to the experimental group which used the recordings even though the experimental group had a mean I. Q. 3.5 points greater than that of the control group. This could hardly be attributed to a superior achievement in arithmetic, since the superiority was only two months and the statistical significance of this superiority was somewhat low. The lack of superiority of the experimental group may in part be attributed to the fact that the use of recordings as instructional devices was new to the pupils of Rea and Laboratory Schools which comprised the experimental group. Increased familiarity throughout the successive parts with the teaching media would be of more significance to the experimental group than to the control group because the teaching medium of the control group was one with which they were already familiar. Three factors may later have served to affect the achievement of the experimental group. First, improvements in the script of the second presentation would be of more significance in relation to the experimental group because the script was used as a guide, subject to modification by the teacher of the control group, while it was presented verbatim to the students of the experimental group. Second, the fact that the presentation of materials to the control groups was followed by discussion while that to the experimental groups
was not, would tend to accent the role of the recording in
the group in which it was used. Third, matters of timing,
volume control, enunciation, vocabulary, placing of speaker,
condition of playing equipment, and quality of recording were
all noted, with improvements being made where possible.

The second presentation of materials dealt with
division of mixed numbers. Table XI, page 34, shows that
there was a difference of 2.5 between the mean I. Q. of the
control group and the mean I. Q. of the experimental group,
the control group being the superior. There are 86 chances
in 100 that the true difference of the means is greater than
zero. The figure itself, 2.5, is not great. With this
presentation of materials, there is a difference of two
months between the mean grade placement of the control and
experimental groups. As was also the case in Part I, the
statistical significance of this difference is not high.
There are only 57 chances in 100 that the figure .2 is
statistically significant. In this case the advantage in
mean I. Q. score lies with the control group, whereas in
Part I the advantage in I. Q. mean score was with the ex-
perimental group. The advantage in arithmetic achievement,
if of enough value to be important, lies with the experi-
mental group, whereas in Part I the control group was superi-
or in this respect. The superiority in the respective cases
was equal.
Table XI, page 34, shows that the progress test scores, with almost complete statistical significance, indicate an advantage for the control group on both progress tests. It will be noted that the superiority of the control group on Progress Test I in this part is less than the superiority of the control group was in Part I. On Part I the control group had a superior mean score of 4.1 on Progress Test I while the same item in Part II was 2.3. On Progress Test II the control group registered an increase of .5 in difference of mean scores over the difference of mean scores on Progress Test I. This is the only case in which the control group increased the difference in mean score from Progress Test I to Progress Test II.

Data on multiplication of decimals is treated in Table XII, page 35. In this presentation the experimental group had a superiority in mean I. Q. score of 2.1. There were 79 chances in 100 that the difference between the true means was greater than zero. In this presentation the difference in mean I. Q. scores is 2.1 and there are 79 chances in 100 that the difference between the true means is greater than zero. This figure is less than in any other part for this point. The item of difference of grade placement means remains constant at .5 in this part but the statistical significance of the item is increased from 57 to 84 chances.
The advantage is with the control group as was the case in Part I.

In regard to Progress Test I it will be noted that the control group made a score of 12.9 while the experimental group made a score of 10.7. It is true, however, that in this part the superiority of the control group in the matter of difference between mean scores is less than in any previous presentation. On Progress Test II this superiority is practically eliminated, since the difference between the means of the two groups is only .5. The difference of means on Progress Test I is statistically more significant than the same item in Progress Test II, however. There are 99 chances in 100 that the difference between the true means on Progress Test I is greater than zero while the same item on Progress Test II registers only 81 chances in 100. This may be seen in the last column of Table XII, page 35.

There was a difference of 2.0 between the mean I. Q. scores of the experimental and control group in Part IV of the experiment. This difference is in favor of the control group. This is shown in Table XIII, page 36. There were 80 chances in 100 that the difference between the true mean I. Q. scores of the two groups was greater than zero. The difference in mean grade placement scores of the two groups, a somewhat constant factor in this experiment, was one month instead of the usual two. The last column of the table indicates 78
chances in 100 that the difference between the true means is greater than zero.

An examination of the progress test scores in Table XIII, page 36, confirms the trend already indicated. The superiority of the control group in the matter of differences in mean scores on Progress Tests I decreased successively in Parts I, II, and III. Part IV indicates an advantage of 1.8 in terms of mean score on Progress Test I for the experimental group. Although the statistical significance of this item is only 84 chances in 100 as compared with 99 and 100 chances in 100 in a previous presentations, a superiority is indicated. A difference of means of .9 on Progress Test II tends to confirm the advantage gained by the experimental group on Progress Test I.

In considering the point just presented, it should be remembered that in each presentation the intelligence quotients of the control and experimental groups were remarkably similar considering the fact that none was a select group. This is even more true in regard to achievement as indicated by grade placement figures. Table XIV shows clearly this trend wherein the advantage in the item of differences on mean scores on the progress tests passes from the control group to the experimental group. It should also be noted that on Parts I and III, the experimental group made large gains in terms of mean score,
<table>
<thead>
<tr>
<th>Parts</th>
<th>Progress Test I Difference of means in favor of:</th>
<th>Progress Test II Difference of means in favor of:</th>
<th>Loss in terms of mean scores</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>Control Group</td>
<td>Experimental Group</td>
<td>Control Group</td>
</tr>
<tr>
<td>I</td>
<td>4.1</td>
<td></td>
<td>2.1</td>
</tr>
<tr>
<td>II</td>
<td>2.4</td>
<td></td>
<td>2.6</td>
</tr>
<tr>
<td>III</td>
<td>2.2</td>
<td></td>
<td>0.5</td>
</tr>
<tr>
<td>IV</td>
<td>1.7</td>
<td></td>
<td>0.5</td>
</tr>
</tbody>
</table>
thereby reducing the advantage of the control group. On Part II the gain of the control group was only .2. On Part IV the experimental group did not maintain its margin of superiority, the control group reducing the margin of superiority by .9. In this sense, superiority lay in the ability of the experimental group to decrease the advantage of the control group by reducing the difference of the means of the progress tests.

FINDINGS THROUGH CORRELATIONS

Through an examination of the differences in the mean scores on the progress tests it was determined that a trend indicating superiority of the experimental method existed. Statistical treatment of the differences of the means indicated their statistical significance. To further determine the intrinsic nature of this trend certain coefficients of correlation were computed. These computations included (1) the correlation between I. Q. and scores on Progress Test I for the control groups, (2) the correlation between I. Q. and scores on Progress Test I for the experimental group, (3) the correlation between I. Q. and scores on Progress Test II for the control groups, and (4) the correlation between I. Q. and scores on Progress Test II for the experimental groups. In the same way, the coefficients of
correlation between grade placement and scores on Progress Tests I and II for both the control and experimental groups on each part were computed.

As has been stated, these correlations were expected to reveal something of the intrinsic nature of the trend indicating the superiority of the method used with the experimental group. An examination of Table XV reveals the following pertinent facts.

The coefficients of correlation between I. Q. and Progress Test I scores for the control group consistently decreased in each successive presentation. It will be remembered that the superiority of the method used with the control group was less evident with each presentation when interpreted in terms of mean progress test scores. If the same group had been used as the control group each time this decrease might be attributed to the fact that the method used failed to consistently challenge the students. Since the groups were alternated this cannot be said to be true.

The coefficients of correlations on this same item with the experimental group showed no consistent trend but with one exception were higher than those of the control group. This exception occurred in the first part. The coefficient of correlation between I. Q. and Progress Test I scores for the control group on part IV showed a minimum relationship,
### Table XV

**Coefficients of Correlation for Major Factors, Both Groups**

<table>
<thead>
<tr>
<th>Part</th>
<th>Item</th>
<th>Control Group</th>
<th>Experimental Group</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td></td>
<td>P. T. I</td>
<td>P. T. II</td>
</tr>
<tr>
<td>I</td>
<td>I. Q.</td>
<td>0.668 ± 0.067</td>
<td>0.568 ± 0.049</td>
</tr>
<tr>
<td>I</td>
<td>G. P.</td>
<td>0.608 ± 0.056</td>
<td>0.518 ± 0.040</td>
</tr>
<tr>
<td>II</td>
<td>I. Q.</td>
<td>0.558 ± 0.038</td>
<td>0.687 ± 0.058</td>
</tr>
<tr>
<td>II</td>
<td>G. P.</td>
<td>0.637 ± 0.050</td>
<td>0.608 ± 0.045</td>
</tr>
<tr>
<td>III</td>
<td>I. Q.</td>
<td>0.507 ± 0.029</td>
<td>0.375 ± 0.016</td>
</tr>
<tr>
<td>III</td>
<td>G. P.</td>
<td>0.497 ± 0.028</td>
<td>0.375 ± 0.016</td>
</tr>
<tr>
<td>IV</td>
<td>I. Q.</td>
<td>0.323 ± 0.013</td>
<td>0.167 ± 0.003</td>
</tr>
<tr>
<td>IV</td>
<td>G. P.</td>
<td>0.333 ± 0.102</td>
<td>0.251 ± 0.008</td>
</tr>
</tbody>
</table>
while the same item for the experimental group maintained medium significance.

The coefficients of correlation between I. Q. and the scores on Progress Test II for the control group decreased with each presentation except Part II. The same is true of the experimental group, only the exception occurred in Part III. The trend is influenced by the fact that on Parts III and IV for the control group the coefficients of correlation were too low to be significant. This is particularly true of Part IV. In the experimental group, Parts III and IV, the coefficients of correlation for this same item were of low medium significance, .436 and .497.

The coefficients of correlation between grade placement and the scores on Progress Test I for the control group decreased in each successive presentation except Part II. The coefficients of correlation were of distinct medium significance on Parts I and II; Part III was medium; while Part IV was doubtful. The same item for the experimental group showed no consistent trend through the successive presentations. In two parts the coefficients of correlation in respect to this item
in the control group are much greater while in the other
two parts they are much less. Part III of the experimental
group showed a coefficient of correlation between grade
placement and scores on Progress Test I of only .393.

The coefficients of correlation between grade
placement and scores on Progress Test II for the control
group decreased with each successive presentation except
one, Part II. Parts III and IV showed coefficients of
correlation of only .375 and .251 respectively. Although
the coefficients of correlation between grade placement
and scores on Progress Test II for the experimental group
showed no trend, they are so much greater than the same
item for the control group that the differences become
significant. Two of the highest coefficients of the
entire project are to be found in this area. This shows
that upon the second presentation of instructional
materials in each instructional area the pupils using
the experimental method came closer to making scores
approximating their normal arithmetic achievement than
at any other time.

Said: In each part for the control groups except Part II
the correlation between I. Q. and progress test scores is
greater for Progress Test I than for Progress Test II. The same is true for the experimental group, only the exception occurs in Part I. With the exceptions of Parts III and IV for the control group, the difference between coefficients of correlation for I. Q. and Progress Test I scores and I. Q. and Progress Test II scores was about .10. Exclusive of the exceptions noted, the remainder of the coefficients of correlation in this area were of medium significance except the I. Q.--Progress Test II correlation of the experimental group.

The coefficients of correlation between grade placement and progress test scores for the control group was consistently greater for Progress Test I than for Progress Test II. The coefficients of correlation for the same item for the experimental group is consistently greater for Progress Test II than Progress Test I. The control group apparently came nearer making scores approximating their general arithmetic ability upon tests following the first presentation of instructional materials in any one instructional area while the experimental group achieved this degree of perfection upon tests following the second presentation of materials. The two groups might
seem equal with one achieving a greater correlation in respect to grade placement on Progress Tests I while the other achieved the greater correlation on Progress Tests II. This seeming equality is nullified by the fact that in their respective fields of superiority the coefficients of correlations of the experimental group are of much greater value with one exception than those of the control group.

General facts to be observed in Table XV, page 45, include the following. The majority of the coefficients of correlation are of medium or high medium significance. The lowest coefficients of correlation are to be found in Parts III and IV for the control group in all items. These low coefficients of correlation of the control group in these presentations may explain the advantage of the experimental group in mean scores on this particular part. On the other hand the average coefficients of correlation of the experimental group did not show enough of an increase in the successive presentations to indicate that the superiority of this group in the matter of mean scores on Part IV was due to increased employment of the factors measured by an intelligence or achievement test.

It is true, however, that the experimental group generally maintained their coefficients of correlation in these matters through the successive parts while the control group
did not. This would seem to indicate that the lack of
superiority of the control group was due, to a degree, to
the failure of that group to maintain achievement in accord
with intelligence and arithmetic background. This trend may
be noted in Part III. Isolation of the causes of this lag
in the achievement of the control group or determination of
the exact degree to which it is a factor in contributing to
the experimental group's superiority in Part IV would be
most difficult. To say that the lack of superiority of the
control group as indicated by difference of mean scores on
Progress Tests, Part IV, is due entirely to the low corre-
lations between the progress test scores and grade placement
and progress test scores and I. Q. of the group would probably
be a mis-statement of fact. On the other hand, a claim that
the experimental method's superiority in terms of correlations
lies in the fact that over a number of presentations, groups
using this method maintain a higher correlation in respect
to intelligence and arithmetic achievement than as groups
using the more conventional methods remains unproved inasmuch
as an insufficient number of presentations have been made to
prove this true.
CHAPTER IV

SUMMARY, CONCLUSIONS AND RECOMMENDATIONS

I SUMMARY

It was the purpose of this experiment to determine the effectiveness of teaching certain processes in elementary arithmetic through the use of instructional recordings as compared with customary instructional procedures.

The data were collected mainly from the Wm. Rea and Laboratory Schools in Terre Haute, Indiana, and from Glenn School in Vigo County, Indiana. A fourth school, Maryland School, participated in a part of the experiment. The plan of procedure was to establish two groups, a control and an experimental group. Four arithmetic processes were presented with each group of pupils established as the control group two times and as the experimental group two times. Instructional materials were presented to the experimental group each time through the use of recordings and instruction sheets only. The teacher of the control group was provided with a script of the recorded materials adapted for classroom presentation. The teacher was free to modify the script to fit her own particular method. Materials presented to the experimental group through instruction sheets were presented on the chalkboard to the control group. In addition, the
control group was allowed to ask questions in order that the method might more nearly simulate regular classroom procedures.

All pupils taking part in the experiment were first given the Otis Group Intelligence Scale, Advanced Examination: Form A. Then they were given the Stanford Achievement Test, Intermediate Arithmetic Test: Form D. In presenting materials in any one part the pupils were first given a survey test to determine which ones should be eliminated because of acquaintance with the materials to be presented. Next the instructional materials were presented, either through recordings or through the customary teaching procedure. Following this Progress Test I was given to determine the achievement of the group. Next the materials were presented again as in the first presentation. Following this second presentation a second progress test was given. This was different from Progress Test I although materials covered on the two were similar. These tests were prepared by the writer. It should also be noted that examples of materials discussed were placed on instruction sheets and given to the group listening to the recordings. These same examples of materials given to the control group were placed on the chalkboard.

1. The following points are a summary of the findings:

1. On Part I, cancellation of fractions, the control group had a mean I. Q. of 101. The experimental group had
a mean I. Q. of 104.5. There is a difference of 3.5 and there are 88 chances in 100 that the difference of the true means is greater than zero. The mean grade placement of the control group was 5.5, while that of the experimental group was 5.3. There is a difference of .2 in favor of the experimental group and there are 57 chances in 100 that the difference of the true means is greater than zero.

2. The mean score of the control group on Progress Test I was 11.8, while that of the experimental group was 7.7. There is a difference of 4.1 and this figure has complete statistical significance. There is a coefficient of correlation of \( .668 \pm .067 \) between I. Q. and scores on Progress Test I for the control group. For the experimental group the coefficient of correlation is \( .538 \pm .030 \). There is a coefficient of correlation of \( .608 \pm .056 \) between grade placement and scores on Progress Test I for the control group. The coefficient of correlation for the experimental group in this item is \( .467 \pm .023 \).

3. The mean score of the control group on Progress Test II was 11.7, while that of the experimental group was 9.6. There is a difference of 2.1 and there are 96 chances in 100 that the difference of the true means is greater than zero. There is a coefficient of correlation of \( .568 \pm .049 \) between I. Q. and Progress Test II scores for the control group. For the experimental group this coefficient of correlation was
There is a coefficient of correlation of .518 ± .040 between grade placement and Progress Test I scores for the control group. The coefficient of correlation for the experimental group in this item is .508 ± .027.

4. On Part II, division of mixed numbers, the mean I. Q. for the control group was 103.5. This figure for the experimental group was 101. The difference of 2.5 had a statistical significance of 86 chances in 100. The mean grade placement of the control group was 5.1. The same figure for the experimental group was 5.3. The difference of the means is .2. There are 57 chances in 100 that the difference of the true means is greater than zero.

5. The mean score of the control group on Progress Test I was 8.5. That of the experimental group was 6.2. The difference was 2.3. There were 99 chances in 100 that the difference of the true means is greater than zero. There is a coefficient of correlation of .558 ± .038 between I. Q. and Progress Test I scores for the control group. The coefficient of correlation for the experimental group in this item is .628 ± .042. There is a coefficient of correlation of .637 ± .050 between grade placement and Progress Test I scores for the control group. For the experimental group this coefficient of correlation is .707 ± .054.

6. The mean score of the control group on Progress
Test II was 9.0. That of the experimental group was 6.4. The difference of 2.6 has almost complete statistical significance. There is a coefficient of correlation of .687 ±.058 between I. Q. and Progress Test II scores for the control group. This same item for the experimental group is .546 ±.032. The coefficient of correlation between grade placement and Progress Test II scores is .608 ±.045. For the experimental group this coefficient is .804 ±.070.

7. In Part III, multiplication of decimals, the control group had a mean I. Q. score of 104.3. That of the experimental group was 106.4. There is a difference of 2.1. There are 79 chances in 100 that the difference of the true means is greater than zero. The grade placement of the control group was 5.6 while that of the experimental group was 5.4. There were 84 chances in 100 that the difference of the true means is greater than zero.

8. On Progress Test I the control group had a mean score of 12.9 while that of the experimental group was 10.7. The difference is 2.2. This difference had almost complete statistical significance. The coefficient of correlation between I. Q. and Progress Test I scores for the control group is .507 ±.029. The same item for the experimental group is .528 ±.035. The coefficient of correlation between grade placement and Progress Test I scores for the
control group was .497 ± .028. For the experimental group this coefficient was .393 ± .020.

9. The mean score of Progress Test II for the control group, Part III, was 13.8. That of the experimental group was 13.3. The difference is .5. There are 81 chances in 100 that the difference of the true means is greater than zero. The coefficient of correlation between I. Q. and Progress Test II for the control group is .375 ± .016 while the coefficient for the experimental group in this same item is .436 ± .024. The coefficient of correlation between grade placement and Progress Test II scores is .375 ± .050 for the experimental group.

10. On Part IV, division of decimals, the mean I. Q. of the control group was 105.6. That of the experimental group was 103.6. The difference is 2.0. There are 80 chances in 100 that the difference of the true means is greater than zero. The grade placement of the control group was 5.4. That of the experimental group was 5.5. There were 78 chances in 100 that the difference of the true means was greater than zero.

11. On Progress Test I the mean score of the control group was 8.5. That of the experimental group was 10.3. There is a difference of 1.8. There are 84 chances in 100 that the difference of the true means is greater than zero.
The coefficient of correlation between I. Q. and Progress Test I scores for the control group is .323 ± .013. The coefficient for the experimental group in the same item is .548 ± .033. The coefficient of correlation between grade placement and Progress Test I scores for the control group is .333 ± .102. For the experimental group this coefficient is .528 ± .031.

12. On Progress Test II the control group had a mean score of 10.2. That of the experimental group was 11.1. The difference is .9. There are 85 chances in 100 that the difference of the true means is greater than zero. The coefficient of correlation between I. Q. and Progress Test II scores for the control group is .167 ± .003 and that of the experimental group is .497 ± .027. The coefficient of correlation between grade placement and Progress Test II scores for the control group is .251 ± .008 while that of the experimental group in the same item is .717 ± .057.

II. CONCLUSIONS

1. In three of the four parts of this experiment the mean progress test scores of the control group were greater than those of the experimental group. In each successive part, however, its margin of superiority decreased.

2. This establishes a trend toward a lack of
superiority of the control group when value is interpreted in terms of mean scores of these particular tests.

3. This trend is somewhat nullified by low correlations between ability and achievement for the control group in those areas where a lack of superiority of the control group is most prominent.

4. On the other hand, the validity of this trend is substantiated by the fact that its existence was evident throughout the experiment. Consistency also added to the validity of the trend.

5. Correlation figures indicate that the control group scored an achievement rating more nearly in line with their abilities after the first presentation of materials in the various parts. The experimental group developed a higher achievement rating after the materials had been presented a second time. Even so, the correlation between achievement on the progress tests and abilities for the control group after the first presentation was not so great as that of the experimental group after the second presentation.

6. Correlation figures further indicate that the experimental group maintained achievement scores on a level with their abilities better than the control group did. This is true of both progress tests. These correlations showed that in most cases a medium relationship between achievement and abilities existed.
III. RECOMMENDATIONS

In light of the writer's experiences in conducting this experiment, certain recommendations may be made.

1. In order to test the importance of voice and enunciation a trained speaker might be used in making the recordings. An expert radio announcer might well serve in this type of experiment.

2. In light of the success of repetitions with the experimental group in this experiment, multiple repetitions could be tried.

3. A training period during which the pupils of the experimental group could become accustomed to the listening technique required for recordings might serve to eliminate the lag in achievement which the experimental groups exhibited in the first presentations in this experiment.

4. In this experiment a trend was established. However, there were only four parts to this experiment. In an experiment involving a greater number of parts or presentations, the consistency of this trend could be tested.

5. An experiment in which the retention of facts and concepts by each group were tested at intervals would be of value.

6. Experiments applying the technique of recorded instructions to other fields of work and to broader instructional
units would do much to establish its value.

7. An experiment might be established in which the repetitions would be controlled by the individual students.

8. Of major importance would be an experiment in which the recordings as well as the conventional method of teaching, were followed by discussion.

9. Variations in length of time required to play a recording, determination of volume by scientific method, use of more than one speaker, pupil operated apparatus, and fatigue conditions accompanying this method should undoubtedly be matters of further investigation.

It should be clear, of course, to anyone interested in further research on this topic that many of the recommendations listed here could be combined into a single experiment.
BIBLIOGRAPHY

BOOKS

APPENDIX
HOW TO CANCEL
(SURVEY TEST)

Solve these examples. Cancel if you know how.

1. \( \frac{2}{5} \times \frac{5}{9} = \)

2. \( \frac{6}{7} \times \frac{1}{6} = \)

3. \( \frac{5}{8} \times \frac{2}{3} = \)

4. \( \frac{3}{4} \times \frac{7}{15} = \)

5. \( \frac{9}{16} \times \frac{2}{3} = \)
HOW TO CANCEL
(STUDY SHEET)

A. $\frac{1}{5} \times \frac{5}{8} = \frac{5}{40} = \frac{1}{8}$

B. $\frac{1}{5} \times \frac{1}{8} = \frac{1}{8}$

C. $\frac{1}{5} \times \frac{3}{4} = \frac{12}{20} = \frac{3}{5}$

D. $\frac{1}{2} \times \frac{3}{4} = \frac{3}{8}$

E. $\frac{1}{2} \times \frac{2}{5} = \frac{2}{10} = \frac{1}{5}$

F. $\frac{1}{2} \times \frac{2}{5} = \frac{1}{5}$

G. $\frac{2}{3} \times \frac{1}{3} = \frac{2}{24} = \frac{1}{8}$

H. $\frac{3}{8} \times \frac{1}{3} = \frac{3}{24} = \frac{1}{8}$

I. $\frac{2}{3} \times \frac{5}{6} = \frac{10}{36} = \frac{1}{3}$

J. $\frac{2}{3} \times \frac{1}{6} = \frac{2}{18} = \frac{1}{9}$

K. $\frac{1}{3} \times \frac{3}{6} = \frac{1}{2}$

L. $\frac{2}{3} \times \frac{1}{4} = \frac{2}{12} = \frac{1}{6}$

M. $\frac{1}{2} \times \frac{3}{4} = \frac{3}{8}$
HOW TO CANCEL (RECORDING)

This recording is intended to help you in learning how to cancel. Cancellation is used when multiplying or dividing fractions so that the answer will not have to be reduced.

Page one: Notice the problem by capital letter A on the paper before you. Multiply the numerators: one times five equals five. Multiply the denominators: five times eight equals forty. Thus you have the numerator five over the denominator forty, or five-fortieths. Five-fortieths can be reduced by dividing both the numerator and the denominator by five. Five is contained in the numerator five, one time. Thus, one is the final numerator. Five is contained in the denominator forty, eight times. Thus, eight is the final denominator. With a numerator of one and a denominator of eight, the final product is one-eighth. There is a shorter way to work this problem.

Notice the problem beside capital letter B. It is the same as problem A. It is worked differently, however, because cancellation is used. There is a number five in the numerator of one of the fractions. There is a number five in the denominator of one of the fractions. By dividing the numerator five and the denominator five by the same number, the numbers
in the problem are made smaller. Thus, the multiplication is made easier. Divide: five is contained in the numerator five, one time. The numerator five has been crossed out and the new numerator one has been placed just above it. Five is contained in the denominator five, one time. The denominator five has been crossed out and the new denominator one written just below it. Now multiply the numerators: one times one equals one. The product one has been placed in the numerator. Now, multiply the denominators: one times eight equals eight. The product eight has been placed in the denominator. The product is one-eighth. Problems A and B are the same. In problem A the answer five-fortieths must be reduced to one-eighth. In problem B the product does not have to be reduced. Cancellation saves that work.

Problem C has been worked without cancelling. The numerators four and three have been multiplied. Three times four equals twelve. The denominators four and five have been multiplied. Five times four equals twenty. The numerator twelve with the denominator twenty makes the fraction twelve-twentieths. Twelve-twentieths can be reduced to lower terms. What number is contained in twelve, three times? What number is contained in twenty, five times? Yes, it is four. Dividing each the numerator twelve and the denominator twenty by four gives the final product three-fifths.
Problem D is the same as problem C except that it has been cancelled. Notice that one number in the numerator and one number in the denominator have each been cancelled. What number is contained evenly in the numerator four and denominator four? PAUSE Yes, four is contained evenly in each the denominator and numerator one time. The numbers which were cancelled have been crossed out and a new numerator one and a new denominator one have been properly placed. The numerators one and three have been multiplied to get the numerator three for the product. The denominators five and one have been multiplied to get the denominator five for the product. The product is three-fifths. The product doesn't have to be reduced. Cancellation saves that work.

Problem E, like problem A and C, has been worked without cancellation. The product two-tenths is the product of one times two, the numerators, and two times five, the denominators. The product, two-tenths, has been reduced to one-fifth by dividing both the numerator and denominator by two. Two was used, of course, because it was contained evenly in the numerator and the denominator.

Problem F has not been worked. Notice the problem carefully. Is there any number which is contained evenly in one number of the numerator and one number of the denominator? PAUSE Yes, two is contained in the numerator two,
one time and in the denominator two, one time. Cross out the numerator two and place a one above it. Cross out the denominator two and place a one below it. You have cancelled in this problem and are ready to multiply. Multiply the numerators: one times one equals one. Multiply the denominators: one times five equals five. With the numerator one and the denominator five, the product is one-fifth. Now, what was done in this problem? Listen. First, a number was chosen which was contained evenly in one of the numerators and in one of the denominators. Second, this number was then divided into the numerator and into the denominator containing it evenly. The numerator and the denominator used in the division were then crossed out and the new numerator and the new denominator properly placed for multiplying. Third, the numerators were multiplied to get a new numerator; the denominators were multiplied to get a new denominator. The answer did not have to be reduced.

Problem G has been worked without cancellation. The product, three twenty-fourths, had to be reduced to one-eighth.

Notice problem H. It is the same as problem G. Do you see two numbers, one in the numerator and one in the denominator, which might be divided by the same number? The numerator three and the denominator three may each be divided evenly by three. Three is contained in the numerator
three, one time. Cross out the numerator three and place a one above it. Three is contained in the denominator three, one time. Cross out the denominator three and place a one below it. Multiply the numerators. The product is one. Place a one in the position of the numerator in the product. Multiply the denominators. The product is eight. Place an eight in the position of the denominator in the answer. The product is one-eighth.

In problem I the product of the numerators is ten. The product of the denominators is thirty. Ten-thirtieths has been reduced by dividing both the numerator and denominator by ten. Ten was used, of course, because it was contained evenly in the numerator and denominator. The product is one-third.

Problem J is the same as problem I, but it has been cancelled. The numerator five and denominator five each contain five, one time. They have been crossed out and the number one placed in the proper place. The numerators two and one have been multiplied to get the numerator two. The denominators, one and six, have been multiplied to get the denominator six. Two-sixths is the product. What number is contained evenly in two and six? PAUSE Two is contained in two, one time. Two is contained in six, three times. The final answer is one-third. Before leaving this problem notice the two parts of the problem which have not been cancelled. Could they be cancelled?
Notice problem K. It is the same as problems I and J. The numerator five and the denominator five have been cancelled as in problem J. Now, this important question: What number is contained evenly in both the numerator two and the denominator six? Yes, it is two. Two is contained in the numerator two, one time. The two has been crossed out and a one placed above it. Two is contained in the denominator six, three times. The six has been crossed out and a three placed below it. The numerators have been multiplied: one times one equals one. The denominators have been multiplied: one times three equals three. The product is one-third. In this problem all the members of the fractions were cancelled. It is important that all possible numbers be cancelled. Of course, many numbers cannot be cancelled. Two numbers may be cancelled when they contain the same number evenly.

Compare problems L and M. What has been done in problem M that should have been done in problem L? PAUSE Yes, both numerators and both denominators should have been cancelled. The numerator three and the denominator three have been divided by what number? Yes, each contain three, one time. What number is contained evenly in the numerator two and the denominator four? Yes, it is two. Two is contained in the numerator two, one time. Two is contained in the denominator four, two times. These numbers have been properly crossed out in
problem M. By multiplying properly, the product one-half is found.

POINTS TO REMEMBER: To cancel:

First: Find a number in the numerator and a number in the denominator each of which evenly contain some number. They may be divided by that number without any remainder.

Second: Cancel, that is, divide the numbers which you have chosen in the numerator and the denominator by the number which they contain evenly. Cross out as is necessary.

Third: Check carefully to see if you have cancelled all the numbers possible. Be sure to cancel only numerators and denominators. Never cancel two numbers in the numerators or two numbers in the denominators.

Fourth: Multiply the numerators; then multiply the denominators.
To the pupils:

For some time we in this class have been working with multiplication of fractions. We have studied three types of problems in multiplication which deal with fractions. First, we studied problems in which a fraction was multiplied by a whole number as two times one-half. Second, we studied problems in which a whole number was multiplied by a fraction. Third, we studied problems in which a fraction was multiplied by a fraction. Problem A on the board is an example of this kind of problem. Will someone explain Problem A for us?

To the teacher:

The teacher is to call on one of the competent members of the class to explain the problem. If the member of the class doesn't complete the explanation, the teacher should do so. The following should be presented.

To the pupils:

To work this problem, multiply the numerators: one times five equals five. Multiply the denominators: five times eight equals forty. Thus you have the numerator five
over the denominator forty, or five-fortieths. Five-fortieths can be reduced by dividing both the numerator and denominator by five. Five is contained in the numerator five, one time. Thus, one is the final numerator. Five is contained in the denominator forty, eight times. Thus, eight is the final denominator. With a numerator of one and a denominator of eight the final product is one-eighth.

TO THE PUPILS:

In problem A it was necessary to reduce the answer. There is a shorter way to work this problem. Notice this problem.

TO THE TEACHER:

Place this problem on the board;

\[ B. \quad \frac{1}{5} \times \frac{5}{8} = \]

Use the following explanation, cancelling as is usual. The completed explanation and concurrent cancelling will cause the problem to finally look like this:

\[ B. \quad \frac{1}{5} \times \frac{5}{8} = \frac{1}{8} \]
TO THE PUPILS:

It is the same as problem A. It is worked differently, however, because cancellation is used. Cancellation is used in multiplying or dividing fractions so the answer will not have to be reduced. There is a number five in the numerator of one of the fractions. There is a number five in the denominators of one of the fractions. By dividing the numerator five and the denominator five by the same number, the numbers are made smaller and the multiplication is made easier. Divide: five is contained in the numerator five, one time. Cross out the five, and place the new numerator one just above it. Five is contained in the denominator five, one time. Cross out the denominator five, and place the new denominator one just below it. Now multiply the numerators: one times one equals one. Place the product one in the numerator. Multiply the denominators: one times eight equals eight. Place the product eight in the denominator. The product is one-eighth. In problem A the product five-fortieths had to be reduced to one-eighth. In problem B the product does not have to be reduced. Cancellation saves that work.

Problem C is much like problem A. It is correct as worked. Will someone explain the problem for us?
TO THE TEACHER:

The teacher is to call on a competent member of the class to furnish the explanation of problem C. If the member of the class doesn't complete the explanation, the teacher should. The following should be presented.

TO THE PUPILS:

In this problem the four and three have been multiplied. Three times four equals twelve. The denominators four and five have been multiplied. Five times four equals twenty. The numerator twelve with the denominator twenty makes the fraction twelve-twentieths. Twelve-twentieths can be reduced to lower terms. What number is contained in twelve, three times? What number is contained in twenty, five times? PAUSE FOR ANSWER Yes, it is four. Dividing each the numerator twelve and the denominator twenty by four gives the final product three-fifths.

TO THE TEACHER:

Problem D should now be placed on the board and worked by the teacher while the explanation is being given in much the same manner as problem B. Write the problem on the board before the following is read to the pupils.
TO THE PUPILS:

This problem, which we shall call problem D, is the same as problem C. However, we shall cancel in this problem. Notice the problem. What number is contained evenly in the numerator four and in the denominator four? PAUSE FOR ANSWER Yes, four is contained evenly in each the numerator and denominator one time. Cross out the numerator four and place a one above it. Cross out the denominator four and place a one below it. Multiply the numerators: one times three equals three. Multiply the denominators: five times one equals five. The product is three-fifths. The product doesn't have to be reduced. Cancellation saves that work.

Notice problem E. Like problems A and C, it has been worked without cancellation. How is the product two-tenths obtained?

TO THE TEACHER:

Pause for correct answer. If the correct answer is not given, the following should be presented.

TO THE PUPILS:

The product two-tenths is the product of one times two, the numerators, and two times five, the denominators. You have already learned, of course, that two-tenths is reduced
to one fifth by dividing both the numerator and the denominator by two. The two is used because it is contained evenly in both the numerator and denominator.

TO THE PUPILS:

We shall work problem \( F \) together. Notice the problem carefully. Is there any number which is contained evenly in one number of the numerators and one number of the denominators? PAUSE FOR ANSWER Yes, two is contained in the numerator two, one time and in the denominator two, one time. Cross out the numerator two, and place a one above it. Cross out the numerator two, and place a one above it. Cross out the denominator two, and place a one below it. You have cancelled this problem and are ready to multiply. You may do the multiplication for us, _____.

PAUSE WHILE PUPIL COMPLETES WORK Did _____ complete the problem correctly? PAUSE FOR ANSWER

Now, what did we do to solve this example? First, a number was chosen which was contained evenly in one of the numerators and in one of the denominators. Second, this number was divided into the numerator and into the denominator containing it evenly. The numerator and the denominator used in the division were then crossed out and the new numerator and the new denominator properly placed for multiplying. Third, the numerators were multiplied to get a new numerator; the denominators were multiplied to get a new denominator. The answer
did not have to be reduced.

Problem G has been worked without cancellation. The product three-twenty-fourths had to be reduced to one-eighth.

TO THE TEACHER:

Problem H should be placed on the board and worked before the class in much the same manner as problems B and D.

Read ahead.

TO THE PUPILS:

Now I shall place the same fractions on the board as were used in problem G. PLACE PROBLEM ON BOARD This time we shall call them problem H and shall cancel. Do you see two numbers, one in the numerator and one in the denominator, which might be divided by the same number? PAUSE FOR ANSWER Yes, the numerator three and the denominator three may each be divided evenly by three. Three is contained in the numerator three, one time. Cross out the numerator three, and place a one below it. What is the product of the numerators? PAUSE FOR ANSWER What is the product of the denominators? PAUSE FOR ANSWER The answer is one-eighth.

In problem I, what is the product of the numerators before the answer is reduced? PAUSE FOR ANSWER What is the product of the denominators before the answer is reduced?
PAUSE FOR ANSWER  Ten-thirtieths has been reduced by dividing both the numerator and denominator by ten. Ten was used, of course, because it was contained evenly in both the numerator and denominator. The reduced product is one-third.

What difference do you see between problems I and J?

TO THE TEACHER

Pause for the answer to the preceding question. The answer should indicate that the use of cancellation is the main difference between the two problems.

This question should then be added.

TO THE PUPILS:

Will someone explain the cancellation and reducing in problem J?

TO THE TEACHER:

If the pupil does not complete the explanation the following should be presented.

TO THE PUPILS:

The numerator five and the denominator five each contain five, one time. They have been crossed out and the number one placed in the proper place. The numerators two and one have
been multiplied to get the numerator two. The denominators one and six have been multiplied to get the denominator six. Two-sixths is the answer. Two-sixths can be reduced. What number is contained evenly in two and six? PAUSE FOR ANSWER. Yes, two is the number. Two is contained in two, one time. Two is contained in six, three times. The final answer is one-third. Notice that even though this problem was cancelled, the answer had to be reduced. This was necessary because the problem was not cancelled as much as it might have been.

TO THE PUPILS:

Problem K is completed as problem J should have been. Notice this problem. The numerator five and the denominator five have been cancelled as in problem J. Now, this important question: What number is contained evenly in both the numerator two and the denominator six? PAUSE FOR ANSWER. Yes, it is two. Two is contained in the numerator two, one time. Two is contained in the denominator six, three times. The numerator two has been crossed out and a one placed above it. The denominator six has been crossed out and a three placed below it. The numerators have been multiplied: one times one equals one. The denominators have been multiplied: one times three equals three. The product is one-third. In problem K all possible fractions were cancelled while in problem J the two and six were not cancelled. It is important that
all numbers be cancelled if possible. Of course, many numbers cannot be cancelled. Two numbers may be cancelled when they contain the same number evenly.

Now, compare problems L and M. What has been done in problem M that should have been done in problem L?

TO THE TEACHER:

Pause for the answer to the preceding question. The answer should indicate that cancellation is complete in problem M while in problem L cancellation is incomplete.

The following questions should then be asked.

TO THE PUPILS:

Problem L: What number was used in cancelling the numerator three and denominator three? PAUSE FOR ANSWER The answer should be "three".

What number is contained evenly in the numerator two and the denominator four? PAUSE FOR ANSWER

Yes, two is contained in the numerator two, one time. Two is contained in the denominator four, two times. In problem M these numbers have been properly cancelled. The product is one-half.

We have discussed several types of problems in which cancellation is used. Are there any questions which you would
Here are some points to remember about cancelling:

First: Find a number in the numerator and in the denominator which evenly contain the same number. They may be divided by that number without any remainder.

Second: Cancel, that is, divide the number which you have chosen to use in the numerator and denominator by the number which they contain evenly. Cross out as is necessary.

Third: Check carefully to see if you have cancelled all the numbers possible. Be sure to cancel only numerators and denominators. Never cancel two numbers in the numerator or two numbers in the denominators.

Fourth: Multiply the numerators; then multiply the denominators.
### HOW TO CANCEL

**PROGRESS TEST I**

Multiply. Cancel when possible.

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<table>
<thead>
<tr>
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<tbody>
<tr>
<td>1.</td>
<td>( \frac{1}{3} \times \frac{3}{4} = )</td>
<td>9.</td>
<td>( \frac{1}{3} \times \frac{6}{7} = )</td>
</tr>
<tr>
<td>2.</td>
<td>( \frac{2}{3} \times \frac{1}{2} = )</td>
<td>10.</td>
<td>( \frac{4}{5} \times \frac{3}{4} = )</td>
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<td>3.</td>
<td>( \frac{1}{4} \times \frac{2}{3} = )</td>
<td>11.</td>
<td>( \frac{5}{8} \times \frac{4}{5} = )</td>
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<td>4.</td>
<td>( \frac{2}{3} \times \frac{7}{8} = )</td>
<td>12.</td>
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<td>5.</td>
<td>( \frac{4}{5} \times \frac{15}{10} = )</td>
<td>13.</td>
<td>( \frac{2}{5} \times \frac{5}{8} = )</td>
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<td>6.</td>
<td>( \frac{5}{8} \times \frac{4}{5} = )</td>
<td>14.</td>
<td>( \frac{4}{5} \times \frac{5}{12} = )</td>
</tr>
<tr>
<td>7.</td>
<td>( \frac{2}{3} \times \frac{6}{8} = )</td>
<td>15.</td>
<td>( \frac{3}{4} \times \frac{20}{21} = )</td>
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<tr>
<td>8.</td>
<td>( \frac{3}{8} \times \frac{1}{5} = )</td>
<td>16.</td>
<td>( \frac{5}{16} \times \frac{4}{5} = )</td>
</tr>
</tbody>
</table>
HOW TO CANCEL

PROGRESS TEST II

Multiply. Cancel when possible.

1. \( \frac{1}{2} \times \frac{2}{3} = \)

2. \( \frac{3}{5} \times \frac{2}{3} = \)

3. \( \frac{3}{14} \times \frac{7}{8} = \)

4. \( \frac{2}{3} \times \frac{1}{8} = \)

5. \( \frac{5}{9} \times \frac{3}{10} = \)

6. \( \frac{5}{6} \times \frac{8}{15} = \)

7. \( \frac{4}{5} \times \frac{5}{8} = \)

8. \( \frac{1}{2} \times \frac{3}{7} = \)

9. \( \frac{7}{8} \times \frac{2}{3} = \)

10. \( \frac{5}{6} \times \frac{7}{10} = \)

11. \( \frac{5}{6} \times \frac{3}{5} = \)

12. \( \frac{1}{3} \times \frac{6}{7} = \)

13. \( \frac{5}{6} \times \frac{8}{15} = \)

14. \( \frac{5}{8} \times \frac{4}{5} = \)

15. \( \frac{4}{5} \times \frac{15}{16} = \)

16. \( \frac{9}{15} \times \frac{5}{18} = \)
DIVIDING WITH MIXED NUMBERS

SURVEY TEST

This is a test to see if you have learned to divide
with mixed numbers. Solve any of the examples that you can.

1. \[ 3 \frac{5}{6} \div 3 = \]

2. \[ 3 \frac{1}{3} \div 5 = \]

3. \[ 8 \div 1 \frac{2}{3} = \]

4. \[ 6 \div 3 \frac{1}{3} = \]

5. \[ 4 \frac{2}{8} \div 5 = \]
DIVIDING WITH MIXED NUMBERS

STUDY SHEET

1. Part one \( \frac{1}{2} \div 2 = \)
   Part two \( \frac{3}{2} \div \frac{2}{1} = \)
   Part three \( \frac{3}{2} \times \frac{1}{2} = \frac{3}{4} \) ANSWER

1A. \( \frac{1}{2} \div 2 = \) \quad 4. \( \frac{4}{2} \div \frac{5}{1} = \)
   \( \frac{3}{2} \div \frac{2}{1} = \)
   \( \frac{3}{2} \times \frac{1}{2} = \frac{3}{4} \) ANSWER
   \( \frac{1}{5} = \) ANSWER

2. \( \frac{2}{4} \div 6 = \) \quad 5. \( \frac{4}{3} \div 4 = \)
   \( \frac{11}{4} \div \frac{6}{1} = \)
   \( \frac{11}{4} \times \frac{1}{6} = \frac{11}{24} \) ANSWER

3. \( \frac{3}{8} \div 4 = \) \quad 6. \( \frac{4}{1} \div \frac{1}{2} = \)
   \( \frac{27}{8} \div = \)
   \( \frac{4}{1} \div \frac{5}{2} = \)
   \( \frac{27}{8} = \) ANSWER \quad \( \frac{4}{1} \times \frac{2}{3} = 2 \frac{2}{3} \) ANSWER
DIVIDING WITH MIXED NUMBERS (continued)

STUDY SHEET

7. \[ 9 \div 3 \frac{1}{5} = \]
   \[ = \]
   \[ = \text{ANSWER} \]

10. \[ 5 \frac{1}{4} \div 6 = \]
    \[ = \]
    \[ = \text{ANSWER} \]

8. \[ 10 \div 2 \frac{1}{2} = \]
   \[ = \]
   \[ = \text{ANSWER} \]

11. \[ 8 \div 5 \frac{1}{3} = \]
    \[ = \]
    \[ = \text{ANSWER} \]

9. \[ 2 \frac{1}{2} \div 7 = \]
   \[ = \]
   \[ = \text{ANSWER} \]
This recording is intended to help you in learning how to divide with mixed numbers. There are three points which you should remember while listening to this recording:

First: Do not try to work ahead of the material being talked about on the recording even though you know what to do. Do nothing unless you are told to.

Second: When you are told to write an answer or to work a problem on the paper before you, do so at once. Stop your work when the voice on the recording begins again even though you haven't finished the work which you started.

Third: This recording tells about the problems on the paper before you. Point with your pencil to each number on the paper as that number is being talked about.

Remember: Do not work ahead; start and stop work when you are told to do so, and point to the number on the paper which is being talked about.

Notice the paper which you have before you. Number one: The example, one and one-half divided by two. In this example the one and one-half is the dividend. The two is the divisor. The example, one and one-half divided by two, has been re-written. It reads: three-halves divided by two over
one. The dividend and the divisor in the first part of the example have been changed to improper fractions. The mixed number, one and one-half, was changed to three-halves by multiplying the denominator two times the whole number one, then adding the numerator. Two times one equals two. Two plus one equals three. Since two is still used as a denominator, the one and one-half is now three-halves. The whole number two which is the divisor is changed to an improper fraction by placing it over the number one. Thus, two is made into a numerator and one becomes the denominator. Reading the example up to this point we have this: One and one-half divided by two equals three-halves divided by two over one. Remember: The second part of the example is made up by changing the dividend and divisor in the first part of the example to improper fractions. Notice the last part of the example. The dividend three-halves is re-written as three-halves. Thus, no change is made in the dividend. Notice the arrow from the division sign in the second part of the example to the multiplication sign in the third part of the example. The division sign has been changed to a multiplication sign. Notice the arrow pointing from the divisor in the second part of the example to the fraction in the third part of the example. The improper fraction two over one has been changed to make the fraction one-half. The two, which was in the position of
the numerator in part two, has been placed in the position of the denominator in part three. The one, which was the denominator in part two of the example, has been placed in the position of the numerator in part three. This process, whereby the number in the position of the numerator and denominator of the divisor are changed to opposite positions, is called inverting. \textit{I-N-V-E-R-T-I-N-G}

Thus, in the third part of the example the division sign is changed to a multiplication sign and the fraction used as a divisor in the second part of the example is inverted. By multiplying the numerators, three times one, and the denominators, two times two, the answer three-fourths is found. Let us check to see what is done in each part of the example. Part two: Change both the dividend and divisor to improper fractions. Part three: Change the division sign to a multiplication sign, invert the divisor, and multiply. Example one has been re-written as example one A. In example one A there are no arrows, nor are the parts numbered. Study example one A briefly to be sure that you understand that it is exactly like example one. \textit{PAUSE}

Example two: Two and three-fourths divided by six. Two and three-fourths has been changed to the improper fraction eleven-fourths. Multiply the whole number two by the denominator four. \textit{PAUSE} The product is eight. Add the numerator three. Eight plus three equals eleven. The numerator of the
improper fraction is eleven. The denominator is the same as it was in the mixed number. It is four. The divisor six is changed to an improper fraction by placing it over the denominator one. Thus the example reads: Two and three-fourths divided by six over one. Notice the last part of the example. Two things have to be done. The division sign must be changed to a multiplication sign; the divisor six over one must be inverted to read one-sixth. Multiply: One times eleven is eleven; four times six is twenty-four. The answer is eleven-twenty-fourths.

Draw a circle around the dividend two and three-fourths in example two. PAUSE Draw a circle around the eleven-fourths just below the two and three-fourths. PAUSE Think: To change the mixed number two and three-fourths to an improper fraction multiply the whole number two by the denominator four, then add the numerator three. Four times two equals eight. Eight plus three equals eleven. The denominator is four. The denominator times the whole number plus the numerator makes the numerator of the improper fraction.

Draw a circle around the divisor six in example two. PAUSE Draw a circle around the improper fraction six over one just below the divisor six. PAUSE Think: To change a whole number to an improper fraction place the whole number over the number one. Thus, the whole number six becomes the improper
fraction six over one.

Draw a circle around the second division sign in example two. PAUSE Draw a circle around the multiplication sign just below it. PAUSE Always remember to change the division sign to a multiplication sign in the last part of the problem.

Draw a circle around the fraction one-sixth in the last part of the example. PAUSE Draw a line from this circle to the circle which you have already drawn around the improper fraction six over one just above the one-sixth. PAUSE You see that the divisor, which is an improper fraction in the second part of the problem, has been inverted or "turned up side down" in the last part of the example.

Notice example three. Part of the example has not been placed on the paper. Three and three-eighths divided by four equals twenty-seven-eighths divided by blank. What should be done to the divisor four before it is placed in this blank space? PAUSE Yes, it should be changed to an improper fraction. Remember to change whole numbers to improper fractions by placing the whole number over the number one. Here you have the improper fraction four over one. Write the improper fraction four over one in the blank space. Remember to place the four in the position of the numerator and the one in the position of the denominator. PAUSE Now the example reads: Three and three-eighths divided by four equals
twenty-seven-eighths divided by four over one. Notice the last part of the example. What should be done to the division sign? PAUSE Yes, it should be changed to a multiplication sign and written beside the twenty-seven-eighths. Write it.

PAUSE What should be done to the improper fraction four over one before it is written in the last part of the example? PAUSE Yes, it should be inverted to read one-fourth. Write one-fourth in the proper place in the last part of the example. PAUSE Multiply. PAUSE Twenty-seven times one equals twenty-seven. Eight times four equals thirty-two. The answer is twenty-seven thirtyseconds.

Notice example four. Four and one-half divided by five. Change four and one-half to an improper fraction. Remember to multiply the whole number by the denominator and add the numerator. PAUSE The improper fraction is ninehalves. Write ninehalves in the proper place in the second part of the example. PAUSE Now, finish the example remembering two things: First, change the division sign to a multiplication sign. Second, only the divisor is to be inverted. That has already been done in this example. Finish the example now. PAUSE The answer is nine-tenths.

Next work example five remembering these points: First, change both the dividend and the divisor to improper fractions. Second, change the division sign to a multiplication sign in
the last part of the example. Third, invert the divisor and multiply. PAUSE

The example which you have just worked should read like this: Four and three-fifths divided by four equals twenty-three-fifths divided by four over one equals twenty-three-fifths times one-fourth. Multiplying the numerators—twenty-three times one equals twenty-three. Multiplying the denominators—five times four equals twenty. The answer is twenty-three-twentieths. It is important that you change this improper fraction to a mixed number. To do this the denominator is divided into the numerator; twenty into twenty-three. Twenty is contained in twenty-three, one time, with a remainder of three. Thus the answer is one and three-twentieths. Do not forget: If an answer is an improper fraction—the numerator larger than the denominator—it must be changed to a mixed number. To do this, determine the number of times the denominator is contained in the numerator, writing the remainder, if there is any, as a fraction.

All of the examples discussed up to this time have been mixed numbers divided by whole numbers. Notice example six. In it a whole number is divided by a mixed number. The example is worked exactly as the others except that the positions of the whole and mixed numbers are exchanged. The whole number four has been changed to the improper fraction four over one.
The mixed number one and two-thirds has been changed to the improper fraction five-thirds. The division sign has been changed to a multiplication sign. The divisor five-thirds has been inverted to read three-fifths. The fractions have been multiplied as usual. The answer, an improper fraction, has been changed to a mixed number. Twelve-fifths is changed to two and two-fifths by dividing the denominator five into the numerator twelve. Five is contained in twelve, two times with two remainder, or two and two-fifths.

Work problem seven remembering to change the division sign to a multiplication sign. Also remember that the mixed number three and one-fifth must be changed to an improper fraction then inverted. Remember to put your answer in the correct form. PAUSE The answer is two and thirteen-sixteenths.

Now, solve the rest of the examples on the paper before you. Be sure your answers are in the correct form. Those answers which are improper fractions must be changed to mixed numbers and reduced if necessary. You may continue working.
TO THE PUPILS:

Up to the present time this year we have learned to divide certain kinds of fractions. To-day we are going to learn how to divide with mixed numbers. Example one on the board is an example of division with mixed numbers. In this example a mixed number is divided by a whole number. The example one and one-half divided by two. In this example the one and one-half is the dividend. The two is the divisor. The example, one and one-half divided by two, has been rewritten. It reads: three-halves divided by two over one. The arrows show that the dividend one and one-half has been changed to three-halves. The whole number two has been changed to two over one. The dividend and the divisor in the first part of the example have been changed to improper fractions. The mixed number one and one-half was changed to three-halves by multiplying the denominator two times the whole number one, then adding the numerator. Two times one equals two. Two plus one equals three. Since two is still used as a denominator, the one and one-half is now three-halves. The whole number two, which is the divisor, is changed to an improper fraction by placing it over the number one. Thus, two is made into a numerator and one becomes the denominator.
Reading the example up to this point we have this: One and one-half divided by two equals three-halves divided by two over one. Remember: The second part of the example is made up by changing the dividend and divisor in the first part of the example to improper fractions. Notice the last part of the example. The three-halves is re-written as three-halves. Thus, no change is made in the dividend. Notice the arrow from the division sign in the second part of the example to the multiplication sign in the third part of the example. The division sign has been changed to a multiplication sign. Notice the arrow pointing from the divisor in the second part of the example to the multiplication sign in the third part of the example. The improper fraction two over one has been changed to make the fraction one-half. The two, which was in the position of the numerator in part two, has been placed in the position of the denominator in part three. This process whereby the numbers in the position of the numerator and denominator of the divisor are changed to opposite positions is called inverting. I-N-V-E-R-T-I-N-G. Thus, in the third part of the example the division sign is changed to a multiplication sign, and the divisor is inverted. By multiplying the numerators, three times one, and the denominators, two times two, the answer three-fourths is found. Let us check to see what is done in each part of the example. Part one:
Write the problem. Part two: Change both the dividend and the divisor to improper fractions. Part three: Change the division sign to a multiplication sign, invert the divisor, and multiply. Example one has been re-written as example one A. In example one A there are no arrows, nor are the parts numbered. Study example one A briefly to be sure that you understand that it is exactly like example one. PAUSE

TO THE TEACHER:

Answer any questions that may be asked at this point.

TO THE PUPILS:

Example two: Two and three-fourths divided by six. Two and three-fourths has been changed to the improper fraction eleven-fourths. Multiply the whole number two by the denominator four. PAUSE The product is eight. Add the numerator three. Eight plus three equals eleven. The numerator of the improper fraction is eleven. The denominator is the same as it was in the mixed number. It is four. The divisor six is changed to an improper fraction by placing it over the denominator one. Thus the example reads: Two and three-fourths divided by six equals eleven-fourths divided by six-ones. Notice the last part of the example. Two things have to be done. The division sign must be changed to a
multiplication sign. The divisor six-ones must be inverted
to read one-sixth. Will you multiply to show how the answer
eleven twenty-fourths is found, _____?

TO THE TEACHER:

Call for a different volunteer to come to the chalkboard to aid in next step.

TO THE PUPIL AT CHALKBOARD:

Draw a circle around the dividend two and three-fourths
in example two. Draw a circle around the eleven-fourths just
below the two and three-fourths. PAUSE How is the mixed num­
ber two and three-fourths changed to the improper fraction
eleven-fourths?

TO THE TEACHER:

Allow pupils to answer the question just asked. If
they do not make a satisfactory explanation, the following
should be presented.

TO THE PUPILS:

To change the mixed number two and three-fourths to an
improper fraction multiply the whole number two by the denom­
inator four; then add the numerator three. Four times two
equals eight. Eight plus three equals eleven. The denom-
inator is four. The denominator, times the whole number, plus the numerator, makes the numerator of the improper frac-
tion.

TO THE PUPIL AT THE CHALKBOARD:

Draw a circle around the divisor six in example two. PAUSE Draw a circle around the improper fraction six over one, just below the divisor six. PAUSE How is the whole number six changed to an improper fraction?

TO THE TEACHER:

Allow pupils to answer the question just asked. If they do not make a satisfactory explanation, the following should be presented.

TO THE PUPILS:

To change the whole number six to an improper fraction, place the whole number six over the number one. Thus, the whole number six becomes the improper fraction six over one.

TO THE PUPIL AT THE CHALKBOARD:

Draw a circle around the second division sign in example two. PAUSE Draw a circle around the multiplication sign just below it. PAUSE
TO THE PUPILS:

Always remember to change the division sign to a multiplication sign in the last part of the example.

TO THE PUPIL AT THE CHALKBOARD:

Draw a circle around the fraction one-sixth in the last part of the example. PAUSE Draw a line from this circle to the circle which you have already drawn around the improper fraction six over one.

TO THE PUPILS:

You see that the divisor, which is an improper fraction in the second part of the problem, has been inverted or "turned up side down" in the last part of the example.

TO THE TEACHER:

Excuse the pupil who has been assisting at the chalkboard.

TO THE PUPILS:

Notice example three. Part of the example has not been placed on the board. Three and three-eights divided by four equals twenty-seven-eighths divided by blank. What should be done to the divisor four before it is placed in this blank
space?  PAUSE FOR ANSWER  Yes, it should be changed to the improper fraction four over one. Remember to change whole numbers to improper fractions by placing the whole number over the number one. In this example notice that the four is in the position of the numerator and the one is in the position of the denominator.

TO THE TEACHER:

Place the fraction four over one in the proper place.

TO THE PUPILS:

Now the example reads: Three and three-eights divided by four equals twenty-seven-eights divided by four over one. Notice the last part of the example. What should be done to the division sign?  PAUSE FOR ANSWER  Yes, it should be changed to a multiplication sign.

TO THE TEACHER:

Place the multiplication sign in the proper place.

TO THE PUPILS:

What should be done to the improper fraction four over one before it is written in the last part of the example? PAUSE FOR ANSWER  Yes, it should be inverted to read one-fourth.
TO THE TEACHER:

Write the fraction one-fourth in the proper place.

TO THE PUPILS:

You may multiply the fractions for us, _____. PAUSE WHILE WORK IS BEING COMPLETED The answer is twenty-seven-thirty-seconds.

TO THE TEACHER:

Ask pupil to remain at chalkboard to help with next problem.

TO THE PUPILS:

Notice example four. Four and one-half divided by five.
The first thing to be done in this problem is to change four and one-half to an improper fraction. Remember to multiply the denominator times the whole number, then add the numerator.

TO THE PUPIL AT THE CHALKBOARD:

You may change the four and one-half to an improper fraction, _____.

TO THE TEACHER:

Assist the pupil if necessary.
TO THE PUPILS:

Write the nine-halves in the proper place in the second part of the example. Now, finish the example remembering two things: First, change the division sign to a multiplication sign. Second, only the divisor is to be inverted, and that has already been done in this example.

Everyone please watch as ____ finishes dividing the fractions. **PAUSE WHILE WORK IS BEING COMPLETED** The answer is nine-tenths.

TO THE PUPIL AT THE CHALKBOARD:

You are excused, ____

TO THE PUPILS:

In working example five remember these points: First, change both the dividend and the divisor to improper fractions. Second, change the division sign to a multiplication sign in the last part of the problem. Third, invert the divisor and multiply. You may work this example for us, ____

TO THE TEACHER:

Pause while the pupil which has been chosen completes the work. Assist the pupil if necessary. Excuse the pupil. Then give the following instructions concerning the answer.
TO THE PUPILS:

It is important that this improper fraction in the answer be changed to a mixed number. To do this the denominator is divided into the numerator—twenty into twenty-three. Twenty is contained in twenty-three, one time, with a remainder of three. Thus, the answer is one and three-twentieths. Do not forget: If an answer is an improper fraction—the numerator larger than the denominator—it must be changed to a mixed number. To do this determine the number of times the denominator is contained in the numerator, writing the remainder, if there is any, as a fraction.

All of the examples discussed up to this time have involved mixed numbers divided by whole numbers. Notice example six. In it a whole number is divided by a mixed number. The example is worked exactly as the others except that the positions of the whole and mixed numbers are exchanged. The whole number four has been changed to the improper fraction four over one. The mixed number one and two-thirds has been changed to the improper fraction five-thirds. The division sign has been changed to a multiplication sign. The divisor five-thirds has been inverted to read three-fifths. The fractions have been multiplied as usual. The answer, an improper fraction, has been changed to a mixed number.
Twelve-fifths is changed to two and two-fifths by dividing the denominator five into the numerator twelve. Five is contained in twelve, two times with two remainder.

As we solve the rest of the examples on the board there are three things to remember. Change both the dividend and divisor to improper fractions. Change the division sign to a multiplication sign. Invert the divisor and multiply. If the answer is an improper fraction, it must be changed to a mixed number and reduced if necessary.

You may now complete examples seven to eleven on the board. Copy these examples on the paper which you have before you. Be sure your answers are in the correct form. Those which are improper fractions must be changed to mixed numbers and reduced if necessary. You may continue working.
DIVIDING WITH MIXED NUMBERS

PROGRESS TEST I

1. \( \frac{13}{5} \div 5 = \)

2. \( 6 \div 2 \frac{1}{2} = \)

3. \( 4 \frac{2}{5} \div 2 = \)

4. \( 1 \frac{5}{8} \div 5 = \)

5. \( 12 \div 1 \frac{3}{4} = \)

6. \( 2 \frac{1}{3} \div 4 = \)

7. \( 9 \frac{1}{6} \div 10 = \)

8. \( 3 \frac{1}{5} \div 4 = \)
PROGRESS TEST I (continued)

9. \( \frac{1}{4} \div 5 = \)

10. \( \frac{1}{6} \div 5 = \)

11. \( 5 \div 6 \frac{1}{2} = \)

12. \( 12 \div 3 \frac{1}{3} = \)

13. \( \frac{1}{2} \div 6 = \)

14. \( 3 \frac{5}{8} \div 4 = \)

15. \( 90 \div 5 \frac{5}{8} = \)
DIVIDING WITH MIXED NUMBERS

PROGRESS TEST II

1. \(8 \frac{1}{2} \div \frac{1}{2} = \)

5. \(25 \div 1 \frac{2}{3} = \)

2. \(3 \div 3 \frac{1}{3} = \)

6. \(\frac{7}{8} \div 4 \frac{1}{3} = \)

3. \(2 \div 4 \frac{1}{2} = \)

7. \(\frac{7}{8} \div 14 = \)

4. \(\frac{1}{8} \div 8 = \)

8. \(4 \frac{5}{8} \div 5 = \)
PROGRESS TEST II (continued)

9. \( 8 \div 5 \frac{2}{3} = \)

10. \( 3 \frac{1}{8} \div 6 = \)

11. \( 25 \div 1 \frac{1}{3} = \)

12. \( 10 \div 2 \frac{2}{5} = \)

13. \( 4 \frac{2}{3} \div 7 = \)

14. \( 3 \div 3 \frac{3}{5} = \)

15. \( 3 \frac{1}{4} \div 8 = \)
MULTIPLICATION OF DECIMALS

SURVEY TEST

This is a test to see if you can place the decimal point properly in the product of each example. The examples on this page are correct except that the decimal point has not been placed in the product. Do so if you know how.

1. \[ 4.4 \times .3 = 1.32 \]

2. \[ .6 \times .6 = .36 \]

3. \[ 14.3 \times 2.4 = 34.32 \]

4. \[ .04 \times .8 = .032 \]

5. \[ .16 \times .5 = .08 \]
MULTIPLICATION OF DECIMALS

STUDY SHEET

1. \(0.2 \times 0.2 = 0.04\)
2. \(0.02 \times 0.08 = 0.0016\)
3. \(0.08 \times 0.4 = 0.032\)
4. \(0.006 \times 0.018 = 0.000108\)
5. \(0.85 \times 4.25 = 3.6325\)
6. \(0.061 \times 0.2 = 0.0122\)
7. \(0.26 \times 0.14 = 0.0364\)
8. \(2.1 \times 0.03 = 0.063\)
9. \(32 \times 0.6 = 0.048\)

10. \(118 \times 0.229 = 26.972\)
11. \(3 \times 0.225 = 0.675\)
12. \(12.5 \times 0.42 = 5.20\)
13. \(1.25 \times 0.42 = 0.525\)
14. \(0.14 \times 0.2 = 0.028\)
15. \(2.3 \times 0.005 = 0.0115\)
16. \(0.125 \times 0.03 = 0.00375\)
17. \(0.23 \times 0.02 = 0.0046\)
MULTIPLICATION OF DECIMALS

STUDY SHEET (continued)

18. (1A) 
\[
\begin{array}{c}
\cdot 2 \\
\cdot 3 \\
\hline
\cdot 6
\end{array}
\]

19. (3) 
\[
\begin{array}{c}
\cdot 08 \\
\cdot 2 \\
\hline
\cdot 16
\end{array}
\]

20. (4) 
\[
\begin{array}{c}
\cdot 004 \\
\cdot 9 \\
\hline
\cdot 036
\end{array}
\]

21. 
\[
\begin{array}{c}
\cdot 45 \\
\cdot 3 \\
\hline
\cdot 138
\end{array}
\]

22. (7) 
\[
\begin{array}{c}
\cdot 32 \\
\cdot 64 \\
\hline
\cdot 96
\end{array}
\]

23. 
\[
\begin{array}{c}
\cdot 041 \\
\cdot 2 \\
\hline
\cdot 082
\end{array}
\]

24. (9) 
\[
\begin{array}{c}
\cdot 3 \\
\cdot 48 \\
\hline
\cdot 144
\end{array}
\]

25. 
\[
\begin{array}{c}
\cdot 3 \cdot 4 \\
\cdot 2 \\
\hline
\cdot 68
\end{array}
\]

26. (10) 
\[
\begin{array}{c}
\cdot 616 \\
\cdot 672 \\
\hline
\cdot 68992
\end{array}
\]

27. (11) 
\[
\begin{array}{c}
\cdot 3 \\
\cdot 12 \\
\hline
\cdot 387 \\
\cdot 4772
\end{array}
\]

28. 12.9 
\[
\begin{array}{c}
\cdot 3.7 \\
\cdot 903 \\
\hline
\cdot 387 \\
\cdot 4772
\end{array}
\]

29. (14A) 
\[
\begin{array}{c}
\cdot 16 \\
\cdot 48 \\
\hline
\cdot 3
\end{array}
\]

30. 4.5 
\[
\begin{array}{c}
\cdot 002 \\
\cdot 90
\end{array}
\]

31. .02 
\[
\begin{array}{c}
\cdot 4 \\
\cdot 08
\end{array}
\]

32. (16) 
\[
\begin{array}{c}
\cdot 113 \\
\cdot 3 \\
\hline
\cdot 339
\end{array}
\]
MULTIPLICATION OF DECIMALS
(RECORDING)

This recording is intended to help you with multiplication of decimals. There are three points which you should remember while listening to this recording.

First: Do not try to work ahead of the material being talked about on the recording even though you know what to do. Do nothing unless you are told to do so.

Second: When you are told to write an answer or to work a problem on the paper before you, do so at once. Stop your work when the voice on the recording begins again, even though you haven't finished the work which you started.

Third: This recording tells about the problems on the paper before you. Point with your pencil to each number on the paper as that number is being talked about.

Remember: Do not work ahead; start and stop work when you are told to do so, and be sure to point to the number on the paper which is being talked about.

Notice the study sheet before you. There are thirty-two examples given. The first seventeen are complete and correct. They will be explained for you. Following this you will complete the rest of the examples without help.

Example one. Add the column of figures. Two-tenths plus two-tenths plus two-tenths plus two-tenths equals
eight-tenths. How many times has two-tenths been written in the column of figures? PAUSE Yes, it has been written four times. Now notice example one-A. In this example the decimal fraction two-tenths has been multiplied by the whole number four. The product is the same as in example one. In example one A four is used because there were four two-tenths in example one. Two-tenths is used because it is the number appearing four times in example one. Notice that a decimal point has been placed in the product. By multiplying in example one A the answer eight-tenths was obtained more quickly than it was by adding in example one.

Example two. Four times two-hundredths. Four times two equals eight; four times zero equals zero. Notice that the decimal point in this example is two places from the right in the product. Also, notice that there is only one decimal point in this example and that it is also two figures from the right in the number in which it is found.

Example three. Four times eight-hundredths. Four times eight equals thirty-two. The two has been written. Four times zero equals zero. Zero plus the three which was carried equals three. The three has been written. The product is thirty-two-hundredths. How many places from the right is the decimal point in the example? PAUSE Yes, it is two places from the right of the eight in eight-hundredths. How many
places from the right is the decimal point in the product?
PAUSE Yes, it is two places from the right of the two in thirty-two-hundredths. Thus, there is a two place decimal in the example; there is a two place decimal in the product.

Example four. Three times six-thousandths. Three sixes are eighteen. The eight has been written; the one has been carried. Three times zero equals zero. Zero plus one equals one. The one has been written. Three times the last zero equals zero. The decimal point in the example is three places from the right. The decimal point in the product will be three places from the right. Notice that there is no decimal point by the multiplier three.

Example five. Five times eighty-five-hundredths. The example has been multiplied. The figures in the product are five, two and four. The product is not four-hundred and twenty-five, however, because the decimal point changes it. The decimal point in the multiplicand of the example is two places from the right. There is no decimal point in the multiplier. How many places from the right will the decimal point be in the product? PAUSE Yes, it will be two places from the right. The product is four and twenty-five-hundredths.

Example six. The multiplicand in this problem has a decimal point three places from the right. There is no decimal point in the multiplier. How does this affect the answer?
Yes, a decimal point must be placed in the product three places from the right. Notice that the decimal point is in the correct place.

Example seven. This example is multiplied in the same way as any other multiplication problem. The one-hundred and four in the partial product is the product of four and twenty-six. The twenty-six in the partial product is the product of one and twenty-six. Adding the parts of the partial product produces the figures three, six, and four. Now, why has a decimal point been placed in the product between the three and six? The decimal point has been placed in the product between the three and six, two places from the right, because the decimal point in the multiplicand is two places from the right. Notice that there is no decimal point in the multiplier, however.

Example eight: Notice this example. There is a decimal point one place from the right in the product. Why? The decimal point is in this position in the product because the decimal point in the multiplicand is one place from the right.

Example nine. This example is somewhat different from the ones previously studied because the decimal fraction is used as a multiplier instead of being used as a multiplicand. Does the fact that the decimal point is used in the multiplier instead of the multiplicand make any difference regarding the
placing of the decimal point in the product? PAUSE It does not. Notice example eight. It has a one place decimal in the multiplicand and a whole number as a multiplier. Thus it has a one place decimal in the product. Example nine has a one place decimal in the multiplier and a whole number as a multiplicand. Thus it has a one place decimal in the product. Regardless of whether the decimal point is found in the multiplier or in the multiplicand, the number of decimal places in the product depends upon the number of decimal places in the example.

Example ten. Example ten has been properly multiplied. Why is the decimal point in the product placed three places from the right? PAUSE The decimal point is placed three places from the right in the product because it is found three places from the right in the multiplier of the example and there is no decimal point in the multiplicand. Remember, it makes no difference when pointing off in the product whether the decimal point is found in the multiplier or in the multiplicand. The important point is the number of decimal places in the example.

Example eleven. Five-tenths times three-tenths. This is a new kind of example. Here a decimal point is found in both the multiplier and multiplicand. How shall we know the number of decimal places to point off in the product? Here is
the answer to that question. When multiplying decimals, point off as many places in the product as there are in the multiplier and multiplicand together. In example eleven there is a one place decimal in the multiplier and a one place decimal in the multiplicand. A one place decimal times a one place decimal makes a two place decimal in the product. Remember, point off as many places in the product as there are decimal places in the multiplier and multiplicand together. The product in this example is a two place decimal, fifteen-hundredths.

Example twelve. Here again a decimal is found in the multiplier and in the multiplicand. Two decimal mixed numbers are being multiplied. Why are two places pointed off in the product? PAUSE Two places are pointed off in the product because there is a one place decimal in the multiplier and a one place decimal in the multiplicand. One and one are two. There are two decimal places in the product. The product is fifty-two and fifty-hundredths. The example is multiplied in the same way as any other multiplication problem.

Example thirteen. A decimal fraction, forty-two-hundredths, times a decimal mixed number, one and twenty-five-hundredths. The decimal point in the product is four places from the right because it is two places from the right in the multiplier and two places from the right in the multiplicand.
Two plus two are four.

Example fourteen is important. Multiply: Two times four equals eight; two times one equals two. The decimal point in the multiplier is one place from the right. In the multiplicand it is two places from the right. Two plus one equals three, therefore, the decimal point should be three places from the right in the product. But there are only two figures in the product. What should be done when, according to the number of decimal places in the multiplicand and multiplier together, there should be more decimal places in the product than there are figures? Here there should be three decimal places in the product but there are only two figures, a two and an eight. Notice example fourteen A. A zero has been placed at the left of the figures to provide the three places needed to place the decimal point. Remember: When there are not enough figures in the product to provide the correct number of decimal places needed, place as many zeroes as are needed at the left of the product.

Example fifteen has a mixed decimal multiplied by a decimal fraction. There is a one place decimal in the multiplicand two and three-tenths. The multiplier is a three place decimal. One plus three equals four. A three place decimal times a one place decimal must have four places pointed off in the product. Notice the example. Five times
three equals fifteen. The five has been written; the one has been carried. Five times two equals ten. Ten plus one equals eleven. The eleven has been written. Notice this important point. A zero had to be placed at the left of the first figure in the product in order to make four places for the decimal point in the product. Remember: If necessary, place zeroes to the left of the product to provide the proper number of decimal places.

Example sixteen. Three times five is fifteen. The five has been written, the one carried. Three times two equals six. Six plus one equals seven. Three times one equals three. The figures in the product are three, seven, and five. The multiplicand of this example has three decimal places; the multiplier has one. Three and one are four. There must be four decimal places in the product. Since there are only three figures in the product a zero has been placed at the left of the number three in the product. Thus the product has four decimal places as are needed.

Example seventeen. Twenty-three has been multiplied by two to get forty-six. Why has a zero been placed at the left of the four in the product? PAUSE The zero has been placed at the left of the four in the product because three decimal places are needed in the product. Three decimal places are needed in the product because there are two decimal
places in the multiplicand and one decimal place in the multiplier. Two plus one equals three.

Before you continue with the other examples on the paper before you try to recall these facts.

First: Decimals are multiplied in the same manner as whole numbers. In multiplying decimals, however, a decimal point must be placed in the product.

Second: When multiplying decimals point off as many places in the product as there are in the multiplier and multiplicand together.

Third: Decimal points are not always found in both the multiplier and multiplicand. They may be in the multiplicand as in the first eight examples. They may be in the multiplier as in examples nine and ten. In any case, the number of decimal places pointed off in the product is determined by the number of decimal places in the multiplier and multiplicand together.

Fourth: When the total of the number of decimal places in the multiplier and the multiplicand is greater than the number of places in the product, place zeroes at the left of the figures in the product to provide the needed decimal places. This was done in example fourteen A, fifteen, sixteen, and seventeen.

Watch the paper before you. Number one. A whole
number times a one place decimal gives a one place decimal in the product. Number six. A whole number times a three place decimal gives a three place decimal in the product. Number eleven. A one place decimal times a one place decimal gives a two place decimal in the product. Number thirteen. A two place decimal times a two place decimal gives a four place decimal in the product. Number fourteen. A one place decimal times a two place decimal gives a three place decimal in the product. Number sixteen. A one place decimal times a three place decimal gives a four place decimal in the product.

PAUSE Notice example eighteen. Beside the number eighteen is the number one A. This means that if you do not yet understand how to point off the decimal places in example eighteen you are to look at example one A because the two examples are similar. Likewise, if you have trouble with any of the other examples you may refer back to the example suggested for help. Not all of the examples have numbers referring you to another problem for help.

Examples eighteen to thirty-two have been multiplied for you. You are to place the decimal point in the proper place in the product of each example. Finish as quickly as you can, but be sure that the decimal point is in the proper place in each product.
MULTIPLICATION OF DECIMALS

(Teacher's Copy)

To the Pupils:

Up to the present time this year we have worked with decimals in addition and subtraction. To-day we are going to learn how to place the decimal point when multiplying decimals.

Notice the board. I have placed thirty-two examples before you. The first seventeen are complete and correct. They will be explained for you. Following this you will complete the rest of the examples without help.

Example one. Adding the column of figures we find the sum eight-tenths. How many times has two-tenths been written in the column of figures? PAUSE FOR ANSWER Now notice example one A. In this example the decimal fraction two-tenths has been multiplied by the whole number four. The answer is the same as in example one. In example one A four is used because there were four two-tenths in example one. Two-tenths is used because it is the number appearing four times in example one. Notice that a decimal point has been placed in the answer. By multiplying in example one A the eight-tenths was obtained more quickly than it was by adding in problem one.
Example two. Four times two-hundredths. Four times two equals eight; four times zero equals zero. Notice that the decimal point in this example is two places from the right in the product. Also, notice that there is only one decimal point in the example and that it is also two figures from the right in the number in which it is found.

Example three. Four times eight-hundredths. Four times eight equals thirty-two. The two has been written. Four times zero equals zero. Zero plus the three which was carried equals three. The three has been written. The product is thirty-two hundredths. How many places from the right is the decimal point in the problem? PAUSE FOR ANSWER

Example four. Three times six-thousandths. Three sixes are eighteen. The eight has been written; the one has been carried. Three times zero equals zero. Zero plus one equals one. The one has been written. Three times the last zero equals zero. The decimal point in the problem is three places from the right. Notice that there is no decimal point by the multiplier three.

Example five. Five times eighty-five-hundredths.
The example has been multiplied. The figures in the product are five, two and four. The product is not four hundred twenty-five, however, because the decimal point changes it. The decimal point in the multiplicand of this problem is two places from the right. There is no decimal point in the multiplier. How many places from the right will the decimal point be in the product? **PAUSE FOR ANSWER** It will be two places from the right. The product is four and twenty-five-hundredths.

Example six. The multiplicand in this example has a decimal point three places from the right. There is no decimal point in the multiplier. How does this affect the product? **PAUSE FOR ANSWER** Notice that the decimal point is in the correct place.

Example seven. This example is multiplied in the same way as any other multiplication problem. The one hundred and four in the partial product is the product of four and twenty-six. The twenty-six in the partial product is the product of one and twenty-six. Adding the parts of the partial product produces the figures three, six and four. Now, why has a decimal point been placed in the product between the three and the six?
TO THE TEACHER:

If the answer to the above question is not complete the following should be presented.

TO THE PUPILS:

The decimal point has been placed in the product between the three and the six, two places from the right, because the decimal point in the multiplicand is two places from the right. Notice that there is no decimal point in the multiplier.

TO THE PUPILS:

Example eight. Notice this example. There is a decimal point one place from the right in the product. Why? PAUSE FOR ANSWER

Example nine. This example is somewhat different from the ones previously studied because the decimal fraction is used as a multiplier instead of being used as a multiplicand. Does the fact that the decimal point is used in the multiplier, instead of the multiplicand, make any difference regarding the placing of the decimal point in the product? PAUSE FOR ANSWER
TO THE TEACHER:

The pupil would give a negative answer to the question just asked. This further explanation should then be presented.

TO THE PUPILS:

Notice example eight. It has a one place decimal in the multiplicand and a whole number as a multiplier. Thus it has a one place decimal in the product. Example nine has a one place decimal in the multiplier with a whole number as the multiplicand. Thus it has a one place decimal in the product. Regardless of whether the decimal is found in the multiplier or in the multiplicand, the number of decimal places in the product depends upon the number of decimal places in the problem.

Example ten. This example has been properly multiplied. Why is the decimal point in the product placed three places from the right?

TO THE TEACHER:

If the pupil called upon doesn't complete the explanation the following should be presented.
TO THE PUPILS:

The decimal point is placed three places from the right in the product because it is found three places from the right in the multiplier of the example and there is no decimal point in the multiplicand.

TO THE PUPILS:

Remember, it makes no difference when pointing off in the product whether the decimal point is found in the multiplier or in the multiplicand. The important point is the number of decimal places in the problem.

Example eleven. Five-tenths times three-tenths.

This is a new kind of example. Here a decimal point is found in both the multiplier and multiplicand. How shall we know the number of decimal places to point off in the product? Here is the answer to that question. When multiplying decimals, point off as many places in the product as there are in the multiplier and multiplicand together. In problem eleven there is a one place decimal in the multiplier and a one place decimal in the multiplicand. A one place decimal times a one place decimal makes a two place decimal in the product. Remember, point off as many places in the product as there are decimal places in the multiplier and the multiplicand together. The product in this example is a two.
Example twelve. Here again a decimal is found in the multiplier and in the multiplicand. Two decimal mixed numbers are being multiplied. Why are two places pointed off in the product? PAUSE FOR ANSWER Yes, two places are pointed off in the product because there is one place decimal in the multiplier and one place decimal in the multiplicand. One and one are two. There are two decimal places in the product. The product is fifty-two and fifty-
hundredths. The example is multiplied in the same way as any other multiplication problem.

Example thirteen—a decimal fraction, forty-two-
hundredths, times a decimal mixed number, one and twenty-five-
hundredths. The decimal point in the product is four places from the right because it is two places from the right in the multiplier and two places from the right in the multiplicand. Two plus two are four.

Example fourteen is important. Multiply: Two times four equals—PAUSE FOR ANSWER; two times one equals two. The decimal point in the multiplier is one place from the right. Two plus one equals three. The decimal point should be three places from the right in the product, but there are only two figures in the product. What should be done when, according to the number of decimal places in the multiplicand and
multiplier together, there should be more decimal places in
the product than there are figures? Here there should be
three decimal places in the product, but there are only two
figures, a two and an eight. Notice example fourteen A. A
zero has been placed at the left of the figures to provide
the three places needed to place the decimal point. Remember:
When there are not enough figures in the product to provide
the correct number of decimal places needed, place as many
zeros as are needed at the left of the product.

Example fifteen has a mixed decimal multiplied by a
decimal fraction. There is a one place decimal in the
multiplicand two and three-tenths. The multiplier is a three
place decimal. One plus three equals four. A three place
decimal times a one place decimal must have four decimal places
pointed off in the product. Notice the problem. Five times
three equals fifteen. The five has been written; the one has
been carried. Five times two equals ten. Ten plus one equals
eleven. The eleven has been written. Notice this important
point. A zero had to be placed at the left of the first
figure in the product in order to make four decimal places
for the decimal point in the product. Remember: If necessary,
place zeroes at the left of the product to provide the proper
number of decimal places.

Example sixteen. Three times five is fifteen. The
five has been aritten, the one carried. Three times two equals six. Six plus one equals seven. Three times one equals three. The figures in the answer are three, seven, and five. The multiplicand of this example has three decimal places; the multiplier has one. Three and one are four. There must be four decimal places in the product. Since there are only three figures in the product a zero has been placed at the left of the three in the product. Thus the product has four decimal places as are needed.

Example seventeen. Twenty-three has been multiplied by two to get forty-six. Why has a zero been placed at the left of the four in the product?

TO THE TEACHER:

The answer to this question is important. If the answer isn't completed by the pupil the following should be presented.

TO THE PUPILS:

The zero has been placed at the left of the four in the product because three decimal places are needed in the product. Three decimal places are needed in the product because there are two decimal places in the multiplicand and one decimal place in the multiplier. Two plus one equals three.
Before you continue with the other examples on the board try to recall these facts. First, decimals are multiplied in the same manner as whole numbers. In multiplying decimals, however, a decimal point must be placed in the product. Second, when multiplying decimals, point off as many places in the product as there are in the multiplier and multiplicand together. Third, decimal points are not always found in both the multiplier and multiplicand. They may be in the multiplicand as in the first eight examples. They may be in the multiplier as in examples nine and ten. In any case, the number of decimal places pointed off in the product is determined by the number of decimal places in the multiplier and the multiplicand together. Fourth, when the total of the number of decimal places in the multiplier and the multiplicand is greater than the number of places in the product, place zeroes at the left of the figures in the product to provide the needed decimal places. This was done in examples fourteen A, fifteen, sixteen, and seventeen.

Let us briefly review a few of the problems. Number one. A whole number times a one place decimal gives a one place decimal in the product. Number six. A whole number times a three place decimal gives a three place decimal in the product. Number thirteen. A two place decimal times a
two place decimal gives a four place decimal in the product. Number fourteen A. A one place decimal times a two place decimal gives a three place decimal in the product. Number eleven. A one place decimal times a one place decimal gives a two place decimal in the product. Number sixteen. A one place decimal times a three place decimal gives a four place decimal in the product.

Notice example eighteen. Beside the number eighteen is the number one A. This means that if you do not yet understand how to point off the decimal places in example eighteen, you are to look at example one A, because the two examples are similar. Likewise, if you have trouble with any of the other examples you may refer back to the example suggested for help. Not all of the examples have numbers referring you to another example for help.

Examples eighteen to thirty-two have been multiplied for you. You are to place the decimal point in the proper place in the product of each example. Copy these examples on the paper which you have before you. Finish as quickly as you can, but be sure that the decimal point is in the proper place in each product. Remember to copy only examples eighteen to thirty-two. You may start.
MULTIPLICATION OF DECIMALS

PROGRESS TEST I

1. \[
\begin{array}{c}
.2 \\
.3 \\
\hline
.6
\end{array}
\]

2. \[
\begin{array}{c}
.03 \\
.03 \\
\hline
.09
\end{array}
\]

3. \[
\begin{array}{c}
.09 \\
.05 \\
\hline
.45
\end{array}
\]

4. \[
\begin{array}{c}
.004 \\
.003 \\
\hline
.012
\end{array}
\]

5. \[
\begin{array}{c}
.46 \\
.9 \\
\hline
414
\end{array}
\]

6. \[
\begin{array}{c}
.033 \\
.032 \\
\hline
132
\end{array}
\]

7. \[
\begin{array}{c}
.16 \\
.28 \\
\hline
128
\end{array}
\]

8. \[
\begin{array}{c}
2.4 \\
2 \\
\hline
4.8
\end{array}
\]

9. \[
\begin{array}{c}
4 \\
.2 \\
\hline
8
\end{array}
\]

10. \[
\begin{array}{c}
39 \\
3 \\
\hline
117
\end{array}
\]

11. \[
\begin{array}{c}
326 \\
921 \\
\hline
326
\end{array}
\]

12. \[
\begin{array}{c}
16.8 \\
2.5 \\
\hline
840
\end{array}
\]

13. \[
\begin{array}{c}
.433 \\
.26 \\
\hline
2598
\end{array}
\]

14. \[
\begin{array}{c}
.12 \\
.3 \\
\hline
36
\end{array}
\]

15. \[
\begin{array}{c}
.125 \\
.4 \\
\hline
500
\end{array}
\]
MULTIPLICATION OF DECIMALS

PROGRESS TEST II

1. \( \frac{.6}{6} \)

2. \( \frac{.01}{.09} \)

3. \( \frac{.08}{.72} \)

4. \( \frac{.006}{.024} \)

5. \( \frac{.38}{.6} \)

6. \( \frac{.028}{.140} \)

7. \( \frac{.26}{.39} \)

8. \( \frac{6.1}{61} \)

9. \( \frac{3}{.2} \)

10. \( \frac{48}{192} \)

11. \( \frac{642}{1926} \)

12. \( \frac{12.9}{.046} \)

13. \( \frac{.934}{.5604} \)

14. \( \frac{.21}{.04} \)

15. \( \frac{.164}{.5} \)

16. \( \frac{2802}{33624} \)
DIVISION OF DECIMALS

SURVEY TEST

The division has been completed in each of the examples below. You are to place the decimal place in the quotient if you know how to do so.

1. \( .6 \div 1.8 \)

2. \( .16 \div 0.48 \)

3. \( 7.25 \div 43.50 \)

4. \( 26.5 \div 13.25 \)

5. \( 0.9 \div 7.56 \)

6

43.50

13.25

84

7.2

36

36

36
DIVISION OF DECIMALS

STUDY SHEET

1. Divisor ) Dividend
   Quotient

9. 1.3) 5.2
   \[
   \begin{array}{c|c}
   \hline
   5.2 & 4. \\
   \hline
   \end{array}
   \]

2. .6
   \[
   \begin{array}{c|c}
   \hline
   .64 & .64 \\
   \hline
   \end{array}
   \]

3. .6
   \[
   \begin{array}{c|c}
   \hline
   3.2 & 3.2 \\
   \hline
   \end{array}
   \]

4. 1
   \[
   \begin{array}{c|c}
   \hline
   4 & 4 \\
   \hline
   \end{array}
   \]

10. .02) .06
   \[
   \begin{array}{c|c}
   \hline
   3 & 3 \\
   \hline
   \end{array}
   \]

5. 1
   \[
   \begin{array}{c|c}
   \hline
   1 & 1 \\
   \hline
   \end{array}
   \]

11. .07) 4.9
   \[
   \begin{array}{c|c}
   \hline
   7 & 7 \\
   \hline
   \end{array}
   \]

6. 1
   \[
   \begin{array}{c|c}
   \hline
   1 & 1 \\
   \hline
   \end{array}
   \]

12. .25) 22.5
   \[
   \begin{array}{c|c}
   \hline
   22.5 & \text{Remainder} \\
   \hline
   \end{array}
   \]

3. 3
   \[
   \begin{array}{c|c}
   \hline
   3 & 3 \\
   \hline
   \end{array}
   \]

13. .125) 8.75
   \[
   \begin{array}{c|c}
   \hline
   8 & \text{Remainder} \\
   \hline
   \end{array}
   \]

14. .150) 4.50
   \[
   \begin{array}{c|c}
   \hline
   3 & \text{Remainder} \\
   \hline
   \end{array}
   \]

7. 1.1
   \[
   \begin{array}{c|c}
   \hline
   1.1 & 1.1 \\
   \hline
   \end{array}
   \]

15. 45.5) 31.85
   \[
   \begin{array}{c|c}
   \hline
   31 & \text{Remainder} \\
   \hline
   \end{array}
   \]
DIVISION OF DECIMALS

STUDY SHEET (continued)

16. -- 3. .2) .8

17. -- 5. .8) 5. 6

18. -- 6. .5) 35

19. -- 7. .3) 99

20. -- 8. .14) 84

21. -- 9. 2.3) 9. 2

22. -- 10. .03) .09

23. -- 11. .06) 546

24. -- 12A. .55) 38. 50

25. -- 13A. .175) 8. 750

26. -- 14. .225) 900
This recording is intended to help you with division of decimals. There are three points which you should remember while listening to this recording.

First: Do not try to work ahead of the material being talked about on the recording even though you know what to do. Do nothing unless you are told to do so.

Second: When you are told to write an answer or to work an example on the paper before you, do so at once. Stop your work when the voice on the recording begins again even though you haven't finished the work which you started.

Third: This recording tells about the problems on the paper before you. Point with your pencil to each number on the paper as that number is being talked about.

Remember: Do not work ahead; start and stop work when you are told to do so, and be sure to point to the number on the paper which is being talked about.

Notice the study sheet before you. Examples four to eleven are complete and correct. They will be explained for you. Following this, you are to complete the rest of the examples without help.

Number one. Here you find the names of the parts of an example in division. You will need to know the parts
named here. Note carefully the position of the dividend, the divisor, and the quotient.

Number two. Try to decide what the numbers are that are given here before you hear them on the recording. The first is PAUSE six-tenths. PAUSE Sixty-four-hundredths. PAUSE Three and two-tenths. PAUSE Four. Notice that a decimal point placed after a number does not change its value. It is still a whole number. Notice the next number. Has the decimal point placed after the figure four changed the value of the number? PAUSE No, it has not. The number is still twenty-four. The next number is PAUSE six and eighty-three-hundredths. PAUSE Five-thousandths.

Numbers three, three A, three B, and three C are all parts of the same example. By number three we find the example nine-tenths divided by three-tenths. The only difference between this example and any other example in division is that the decimal points cause the numbers to represent parts instead of wholes. Since you already know how to divide whole numbers, we shall make these decimal fractions in example three into whole numbers. To do this we move the decimal point one place to the right in the divisor and one place to the right in the dividend. To avoid having to erase the old decimal point and write a new one when it is moved, we use the caret which you see in example three A. A caret is made like
The decimal point has been moved one place to the right, but instead of erasing the old decimal point and writing a new one the caret has been placed at the right of the divisor three. By doing this, the divisor three is made a whole number. The decimal point to the left of the three is no longer of any value. Since the divisor three-tenths was changed to a whole number the dividend nine-tenths likewise must be changed. Notice that the decimal point in the dividend has been moved one place to the right just as it was moved one place to the right in the divisor. This makes the dividend nine-tenths into a whole number, since the decimal point is of no value when the caret is placed at the right of the figure nine. The decimal point is placed in the quotient just above the caret in the dividend. Notice: First the caret was located in the dividend. Then the decimal point was located in the quotient above the caret in the dividend.

Example three B. By moving the decimal point one place to the right in each the divisor and the dividend in example three A, the dividend and divisor were made into whole numbers. Now in example three B this question may be asked. How many times is three contained in nine? Three is contained in nine, three times. The three is placed in the quotient directly over the nine. The decimal point in the
quotient was located by the use of carets as shown in example three A. Notice that after the division is completed, the decimal point is at the right of the three in the quotient. It is important to remember that the three is a whole number. Thus, it is seen that three-tenths is contained in nine-tenths, three times.

Let us review briefly the steps taken to solve this example. Number three: Write the problem. Number three A: Move the decimal point to the right in the divisor far enough to make it a whole number. A caret is used to indicate the new position of the decimal point. Next, in example three A, the decimal point is moved to the right in the dividend as many places as it was moved to the right in the divisor. A caret was placed in the divisor at the new place for the decimal point. Since the main problem in the division of decimals is the placing of the decimal point in the quotient, you must remember to place the decimal point in the quotient just above the caret in the dividend. This was done in example three A. The example was completed in example three B. Divide as with whole numbers.

Example three C is exactly like three B but is more easily understood. Follow these points: Count the number of decimal places in the divisor. PAUSE There is one. Keep the one in mind. Now move the decimal point that number of places
to the right in the dividend. The caret has been placed in the new position of the decimal point. The decimal point is then placed in the quotient just above the caret in the dividend. Divide as with whole numbers and the example is complete.

Example four. How many decimal places are to be found in the divisor? PAUSE There is a one place decimal in the divisor. Thus, you know that the decimal point in the dividend must be moved one place to the right. This has been done and a caret has been placed in the new position of the decimal point. A decimal point was placed in the quotient just above the caret in the dividend. Dividing: Four is contained in eight two times. Reading the complete problem we find that four-tenths is contained in eight-tenths, two times.

Example five. There is a one place decimal in the divisor. Thus, the decimal point must be moved one place to the right in the dividend. By doing this the dividend is changed from four and eight-tenths to forty-eight. The decimal point in the quotient is just above the caret in the dividend. Dividing: Six is contained in forty-eight, eight times.

Example six. There is a one place decimal in the divisor. Moving the decimal point one place to the right in the dividend changes the dividend from twenty-four-hundredths to two and four-tenths. A caret is placed in the new position
of the decimal point in the dividend. Just above the caret the decimal point is placed in the quotient. Dividing: Four is contained in twenty-four, six times. Here you will notice that the quotient is not a whole number; it is a decimal fraction, six-tenths. However, the example is solved in exactly the same way. The kind of decimal fraction in the quotient doesn't affect the method of solving an example.

Notice example seven. There is a one place decimal in the divisor, so the decimal point is moved one place to the right in the dividend. A caret is placed in this new position of the decimal point between the two sixes. Just above the caret is the position for the decimal point in the quotient. The quotient is one and one-tenth.

Example eight has a two place decimal in the divisor. Therefore, the decimal point in the dividend must be moved to the right two places, a caret inserted in the proper place, and the decimal point placed in the quotient just above the caret in the dividend. Do not forget to move the decimal point to the right in the dividend as many decimal places as there are decimal places in the divisor. If the divisor has two, three, or four decimal places, the decimal point in the dividend must be moved two, three or four places accordingly. Here in example eight, twenty-four-hundredths is contained in forty-eight-hundredths, two times. The example has been completed. PAUSE
Example nine has a one place decimal in the divisor. Notice that the first step in solving these examples is to determine the number of decimal places in the divisor. Since there is a one place decimal in the divisor, the decimal point in the dividend has been moved one place to the right and a caret inserted in this place. Just above this caret the decimal point has been placed in the quotient. Thirteen is contained in fifty-two, four times.

Why has the caret in the dividend of example ten been placed two decimal places to the right of the decimal point? PAUSE The caret has been placed two places to the right of the decimal point in the dividend because there are two decimal places in the divisor of the example. Two-hundredths is contained in six-hundredths, three times.

In example eleven the quotient seven and one-tenth was obtained by dividing four-hundred ninety-seven-thousandths by seven-hundredths. Remember that the caret is placed two places to the right of the decimal point in the dividend because there are two decimal places in the divisor, seven-hundredths.

Example twelve presents a new problem. You have learned to find the number of decimal places in the divisor and then to move the decimal point to the right in the dividend that number of places. Here in example twelve the divisor has two decimal places. Hence, we know that the decimal point in the dividend
should be moved two places to the right. Here is the new problem. How can the decimal point in the dividend be moved two places to the right when there is only one decimal place at the right of the decimal point?

For an answer to this question notice example twelve A. The decimal point has been moved two places to the right in the dividend by placing a zero at the right of the last figure in the dividend. Study the dividends of examples twelve and twelve A carefully to be sure that you understand the placing of the extra zero in the dividend. The division in the example is the same as with whole numbers. Twenty-five-hundredths is contained in twenty-two and fifty-hundredths, ninety times.

Examples thirteen and thirteen A are similar to examples twelve and twelve A. In examples thirteen and thirteen A the divisors have three decimal places, therefore, the decimal points in the dividends must be moved three places to the right. This could not be done in example thirteen. Notice how this was done in example thirteen A. PAUSE A zero was placed at the right of the last figure in the dividend. By doing this the decimal point may be moved three places. The decimal point in the quotient is places over the caret in the dividend. The quotient is seventy. You will notice that in dividing the divisor one-hundred twenty-five was first divided into eight-hundred seventy-five. After multiplying the seven times the
divisor the product was subtracted from the dividend figures used in dividing. No difference was found between the two numbers. The last figure in the dividend was then brought down. Since the divisor is not contained in zero at all a zero was placed in the quotient. Multiplying the zero in the quotient times the divisor, writing the product under the last figure brought down, and then subtracting, completes the example.

Example fourteen. A three place divisor requires that the decimal point in the dividend be moved three decimal places to the right. Notice that the decimal point can be moved the proper number of places in this example without placing any zeros at the right of the last figure in the dividend. By properly placing the caret in the dividend and the decimal point in quotient above the caret, we prepare the example for division. One-hundred-fifty-thousandths is contained in four-hundred-fifty-thousandths, three times.

Example fifteen. There is a one place decimal in the divisor. The decimal point in the dividend has been moved one place to the right. By dividing as with whole numbers the quotient seven-tenths is found.

Examples sixteen to twenty-six have another number by the number of the example. If you don't understand any one of the examples sixteen to twenty-six you are to study the
example which the second number by the example indicates. For example, if you do not understand example sixteen, you are to study example three C. If you do not understand example seventeen, you are to study example five. You may continue. Place the decimal point in the quotients of examples sixteen to twenty-six, but be sure you correctly place the caret in the dividend by first determining the number of decimal places there are in the divisor. You may begin.
DIVISION OF DECIMALS
(TEACHER'S COPY).

TO THE PUPILS:

Up to the present time this year we have learned to add, subtract, and multiply decimals. To-day we are going to learn how to divide decimals.

Notice the examples on the board. Examples four to eleven are complete and correct. They will be explained for you. Following this you are to complete the rest of the examples without help.

Number one. Here you find the names of the parts of an example in division. You will need to know the parts named here. Note carefully the position of the dividend, the divisor, and the quotient.

Number two. Here we find several decimals and whole numbers. Let us be sure we can read these numbers correctly. Will you read these numbers for us, _____?

TO THE TEACHER:

Allow pupil to read numbers, assisting if necessary.

TO THE PUPILS:

Notice that a decimal point placed after a number does not change its value. It is still a whole number. Notice the
numbers four and twenty-four, for example.

Numbers three, three A, three B, and three C are all parts of the same example. By number three we find the example, nine-tenths divided by three-tenths. The only difference between this example and any other example in division is that the decimal points cause the numbers to represent parts instead of wholes. Since you already know how to divide whole numbers, we shall make these decimal fractions in example three into whole numbers. To do this, we move the decimal point one place to the right in the divisor and one place to the right in the dividend. To avoid having to erase the old decimal point and write a new one when it is moved, we use the caret which you see in example three A. A caret is made like a v up side down. Notice the divisor in example three A. The decimal point has been moved one place to the right, but instead of erasing the old decimal point and writing a new one, the caret had been placed at the right of the divisor three. By doing this, the divisor three is made into a whole number since the decimal point to the left of the three is no longer of any value. Since the divisor three-tenths was changed to a whole number, the dividend nine-tenths likewise must be changed. Notice that the decimal point in the dividend has been moved one place to the right, just as it was moved one place to the right in the divisor. What does this do to
the dividend, ______? PAUSE FOR ANSWER

TO THE PUPILS:

The decimal point is placed in the quotient just above the caret in the dividend. Notice: First the caret was located in the quotient above the caret in the dividend.

Example three B. By moving the decimal point one place to the right in each the divisor and the dividend in example three A the dividend and the divisor were made into whole numbers. Now in example three B this question may be asked: How many times is three contained in nine? PAUSE FOR ANSWER

The three is placed in the quotient directly over the nine. The decimal point in the quotient was located by the use of carets as shown in example three A. Notice that after the division is completed the decimal point is at the right of the three in the quotient. Will you read the quotient for us, ______?

TO THE TEACHER:

Let us briefly review the steps taken to solve this example. Number three: Write the problem. Number three A: Move the decimal point to the right in the divisor far enough to make it a whole number. A caret is used to indicate the new position of the decimal point. Next, in example three A, the decimal point is moved to the right in the dividend as
many places as it was moved to the right in the divisor. The caret was placed in the dividend at the new place for the decimal point. Since the main problem in the division of decimals is the placing of the decimal point in the quotient, you must remember to place the decimal point above the caret in the dividend. This was done in example three A. The example was completed in example three B. Divide as with whole numbers.

Example three C is the same as three B but is more easily understood. How many decimal places are there in the divisor, _____? PAUSE FOR ANSWER Keep this answer in mind. Notice that the decimal point has been moved that number of places to the right in the dividend. The caret has been placed in the new position of the decimal point. The decimal point is then placed in the quotient just above the caret in the dividend. Divide as with whole numbers and the example is complete.

Example four. How many decimal places are to be found in the divisor? PAUSE FOR ANSWER Since there is a one place decimal in the divisor, the decimal point in the dividend must be moved one place to the right. This has been done and a caret has been placed in the new position of the decimal point. A decimal point was placed in the quotient just above the caret in the dividend. Dividing: Four is
contained in eight, two times. Reading the complete example we find that four-tenths is contained in eight-tenths, two times.

Example five. There is one decimal place in the divisor. Thus, the decimal point must be moved one place to the right in the dividend. By doing this the dividend is changed from four and eight-tenths to forty-eight. Where is the decimal point placed in the quotient, _____? PAUSE FOR ANSWER

Dividing: Six is contained in forty-eight, eight times.

TO THE PUPILS:

Example six. There is a one place decimal in the divisor. Moving the decimal point one place to the right in the dividend changes the dividend from twenty-four-hundredths to two and four-tenths. A caret is placed in the new position of the decimal point in the dividend. Just above the caret the decimal point is placed in the quotient. Dividing: Four is contained in twenty-four, six times. Will you read the quotient, _____? PAUSE FOR ANSWER Notice that the quotient is not a whole number; it is a decimal fraction, six-tenths. However, the example is solved in exactly the same way. The kind of decimal fraction in the quotient doesn't affect the method of solving an example.

In example seven the divisor is a one place decimal
so the decimal point is moved one place to the right in the dividend. A caret is placed in the new position of the decimal point between the two sixes. Just above the caret is the position for the decimal point in the quotient. The quotient is one and one-tenth.

How many decimal places are there in the divisor of example eight, ____? PAUSE FOR ANSWER How is the new position of the decimal point in the dividend marked, ____? PAUSE FOR ANSWER After the new position of the decimal point in the dividend has been marked by a caret, how is the position of the decimal point in the quotient determined, ____? PAUSE FOR ANSWER Do not forget to move the decimal point to the right in the dividend as many places as there are decimal places in the divisor. If the divisor has two, three, or four decimal places, the decimal point in the dividend must be moved two, three, or four places accordingly. Here in example eight, twenty-four-hundredth is contained in forty-eight-hundredths, two times. The example has been completed.

How many decimal places are to be found in the divisor in example nine, ____? PAUSE FOR ANSWER Notice that the first step in solving these examples is to determine the number of decimal places in the divisor. Since there is a one place decimal in the divisor, the decimal point in the
dividend has been moved one place to the right and a caret
inserted in this place. Just above this caret the decimal
point has been placed in the quotient. Thirteen is contained
in fifty-two, four times.

Why has the caret in the dividend of example ten been
placed two decimal places to the right of the decimal point,
_____?

TO THE TEACHER:

Pause for answer; assist as is necessary.

TO THE PUPILS:

By dividing we see that two-hundredths is contained in
six-hundredths, three times.

In example eleven the quotient seven and one-tenth was
obtained by dividing four-hundred-ninety-seven-thousandths by
seven-hundredths. Remember that the caret is placed two places
to the right of the decimal point in the dividend because
there are two decimal places in the divisor, seven-hundredths.

Example twelve presents a new problem. You have learned
to find the number of decimal places in the divisor and then
to move the decimal point to the right in the dividend that
number of places. Here in example twelve the divisor has
two decimal places. Hence, we know that the decimal point in
the dividend should be moved two places to the right. Here is the new problem. How can the decimal point in the dividend be moved two places to the right when there is only one decimal point?

For an answer to this question notice example twelve A. The decimal point has been moved two places to the right in the dividend by placing a zero at the right of the last figure in the dividend. Study the dividends of examples twelve and twelve A carefully to be sure that you understand the placing of the extra zero in the dividend. The division in the example is the same as with whole numbers. Twenty-five-hundredths is contained in twenty-two and fifty-hundredths, ninety times. Examples thirteen and thirteen A are similar to examples twelve and twelve A. In examples thirteen and thirteen A the divisors have three decimal places. Therefore, the decimal points in the dividends must be moved three places to the right. This could not be done in example thirteen. What was done in example thirteen A that made it possible to move the decimal point in the dividend three places to the right?

TO THE TEACHER:

Pause for answer. Make necessary explanations.
TO THE PUPILS:

The decimal point in the quotient is placed over the caret in the dividend. The quotient is seventy. You will notice that the divisor one-hundred twenty-five was first divided into eight-hundred seventy-five. After multiplying the seven times the divisor the product was subtracted from the dividend figures used in the dividing. No difference was found between the two numbers. The last figure in the dividend was then brought down. Since the divisor is not contained in zero at all a zero was placed in the quotient. Multiplying the zero in the quotient times the divisor, writing the product under the last figure brought down, and then subtracting, completes the example.

Example fourteen. How many places must the decimal point in the dividend be moved to the right? PAUSE FOR ANSWER Is it necessary to place any zeroes at the right of the last figure in the dividend in this example? PAUSE FOR ANSWER We prepare the example for division by properly placing the caret in the dividend and the decimal point in the quotient above the caret. One-hundred-fifty-thousandths is contained in four-hundred-fifty-thousandths, three times.

Example fifteen. Why has the decimal point in the dividend been moved one place to the right? PAUSE FOR ANSWER By dividing as with whole numbers the quotient seven-tenths
Examples sixteen to twenty-six have another number by the number of the example. If you don't understand any one of the examples sixteen to twenty-six, you are to study the example which the second number of the example indicates. For example, if you do not understand example sixteen, you are to study example three C. If you do not understand example seventeen, you are to study example five. You are now to copy examples sixteen to twenty-six, working each example as you copy it. Place the decimal point in the quotients of the examples. Be sure you also correctly place the caret in the dividend by first determining the number of decimal places there are in the divisor. You may begin.
DIVISION OF DECIMALS

PROGRESS TEST I

All except three of the examples on this page have been divided. You are to place the decimal point in the quotient of those examples which have been divided. Solve and place the decimal point in the quotient of those three examples not yet divided.

1. .6) .6 \[ \frac{1}{6} \]

2. .3) 4.8 \[ \frac{1}{6} \]

3. .6) 2.4 \[ \frac{1}{4} \]

4. .4) .44 \[ \frac{11}{44} \]

5. .25) 32.5

6. .2) .08

7. 2.2) 46.2

8. 1.2) 3.60

9. .9) 7.929

10. .07) 1.54

11. 1.6) 6.4

12. .4) .864

13. 1.25) 2.5

14. .02) .08

15. 1.2) 9.6
DIVISION OF DECIMALS

PROGRESS TEST II

All except three of the examples on this page have been divided. You are to place the decimal point in the quotient of those examples which have been divided. Solve and place the decimal point in the quotients of those three examples not yet divided.

1. $1.35)5.4$
2. $3.17)5.1$
3. $0.6)3.6$
4. $1.7)0.77$
5. $0.35)45.5$
6. $0.3)0.09$
7. $4.4)79.2$
8. $0.14)4.20$
9. $0.8)5.288$
10. $0.06)1.98$
11. $1.8)5.4$
12. $0.3)0.693$
13. $1.35)5.4$
14. $0.02)0.06$
15. $0.16)9.6$