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# USING STANDARDIZED TESTS TO IDENTIFY PRIOR KNOWLEDGE NECESSARY FOR SUCCESS IN ALGEBRA: A PREDICTIVE ANALYSIS

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A Dissertation

Presented to

The College of Graduate and Professional Studies

Department of Curriculum, Instruction, and Media Technology

Indiana State University

Terre Haute, Indiana

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In Partial Fulfillment

of the Requirements for the Degree

Doctor of Philosophy

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by

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May 2014



Keywords: algebra readiness, predicting success, prior knowledge, standardized tests, mathematics curriculum

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ABSTRACT

This study sought to determine if there is a relationship between students' scores on the eighth-grade Indiana State Test of Education Progress Plus (ISTEP+) exam and success on Indiana's Algebra End-of-Course Assessment (ECA). Additionally, it sought to determine if algebra success could be significantly predicted by the achievement in one or more of the seven individual reporting subcategories on the ISTEP+ exam. The relationship between the score on the language arts portion of the test and success in algebra was also explored.

Successful completion of algebra and a minimum score on the Algebra ECA is required for high school graduation in the state of Indiana. It is imperative that students master this difficult subject, and educators need to understand how to help all students achieve this goal. This quantitative study utilized regression analyses to determine if the eighth-grade ISTEP+ exam could predict a significant proportion of the variance in Algebra ECA scores. More specifically, multiple regression analysis was utilized to determine if any one of the seven reporting subcategories was a significant predictor of the variance in the algebra scores. If the specific content of one or more reporting subcategories could be linked to algebra success, educators would know where and how to focus instruction and remediation efforts.

Because a review of the literature also revealed a potential link between reading and math scores, a regression analysis was conducted to determine if the eighth-grade ISTEP+ language arts score predicted a significant proportion of the variance in Algebra ECA scores. The study

concluded that the language arts score was a significant predictor, although it did not explain much of the variance in Algebra ECA scores.

For all three models, the scores of students from two different cohorts from the same school district were utilized in the study. The first cohort consisted of students entering ninth grade in the fall of 2010 and the second consisted of students entering ninth grade in the fall of 2011. Each cohort was then divided into an advanced group consisting of students who took both the ISTEP+ and the Algebra ECA in eighth grade and an average group consisting of students who did not take the Algebra ECA until the end of ninth grade. All models in the study proved significant, although there was evidence of multicollinearity and the amount of variance predicted varied greatly.

## ACKNOWLEDGMENTS

Many people supported and encouraged me in my pursuit of this degree, and I could not have achieved this without their help. Dr. Corey and Dr. Brown offered words of advice and encouragement as members of my committee. Dr. Tinnerman, my committee chairperson, further supported me through this process.

My colleagues also offered much support. The superintendent of my school district both encouraged me in this endeavor and gave me permission to use district data in the study. Additionally, I was given flexibility in my schedule to travel to Terre Haute to complete the degree requirements. Fellow teachers and students willingly listened to me talk about my project and how my research affected my teaching.

My friends encouraged me tirelessly throughout this process, helping to hold me accountable with my time and asking for frequent updates.

My family never wavered in their faith in me as I began this journey. My grandparents, Stanley and Marjorie Miller, gave me a quiet place to work on my dissertation out of town and made sure that I stayed well fed. My parents, Herb and Deanna Jensen, provided their continuous support and prayers throughout this process.

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# CHAPTER 1

## INTRODUCTION

Algebra has long been considered a “gatekeeper” subject, the door through which students must pass to gain access to advanced mathematics classes, greater post-secondary educational opportunities, and more lucrative employment. Schoenfeld (1995), working with the Algebra Initiative Colloquium sponsored by the United States Department of Education, described algebra as an “academic passport for passage into virtually every avenue of the job market and every street of schooling” (p. 53). With 28 states requiring exit exams for graduation in 2010 (Dietz, 2010), algebra is no longer merely an integral key to educational advancement; rather, it is a minimum and necessary requirement for high school graduation.

### Statement of the Problem

Numerous studies highlight the social and economic issues associated with failing to earn a high school diploma. High school dropouts reportedly have lower earnings, greater levels of unemployment, and higher incidents of incarceration (Catterall, 1988). Because success in algebra is necessary for graduation, mathematics educators must find a way to help all students become successful in this critical course. Additionally, according to the report released by the National Mathematics Advisory Panel (2008), there is a strong correlation between completing Algebra II and both graduation from college and increased earnings. Algebra success at an earlier age is important so that students can pursue more advanced mathematics courses and open the pathways to the ever expanding job opportunities in science and technology. As Schoenfeld (1995) wrote, “There is a new literacy requirement for citizenship. Algebra today plays the role that reading and writing did in the Industrial Age” (p. 57).

Despite the important role that algebra plays in today’s society, too many students are still failing to earn credit in Algebra I by the end of their ninth-grade year. Additionally, in the state of Indiana, an even greater number are failing to obtain the minimum qualifying score on the Algebra End-of-Course Assessment (ECA). According to the Indiana Department of Education (IDOE, 2013b), only 74% of ninth-grade, first-time testers passed the Algebra ECA in the 2010–2011 school year. The percentage decreases sharply to less than 50% for those students above ninth grade who took the test for the first time.

In this age of increasing accountability, schools are held responsible for the number of students passing the state-mandated standardized tests, such as Indiana’s Algebra ECA. The assumption is that, as scores are published and schools face public scrutiny, reform will occur and schools will improve (Bedwell, 2004). Questioning the ability of such practices to produce more effective schools, Stiggins (1999) referred to this as “intimidation by assessment” (p. 192). Dire consequences await those schools that fail to demonstrate adequate growth in student achievement. More important than the potential fallout for schools and educators is the impact that not passing graduation qualifying exams can and will have on today’s youth. Mathematics educators must seek ways to help students succeed in this critical class and on this high-stakes test, thereby helping them open the door to their futures.

## Theoretical Framework

Rather than viewing these standardized assessments as merely punitive, thorough analysis of the results could actually help inform educational practice. This is the essence of data-driven instruction, using data from assessments such as the state exams to make decisions about effective instruction. Stiggins (1999) argued for increased use of effective classroom assessment, and Bedwell (2004) recognized the potential usefulness of data from statewide tests to identify strengths and weaknesses in teaching and learning, especially when data can be obtained on individual standards as opposed to overall scores. This information can aid educators in creating long-range instructional plans. An analysis of the data could also potentially be used to bridge the gap between middle school mathematics and algebra, helping to determine the necessary prerequisites for algebra success.

Much research has been conducted regarding the cognitive prerequisites for success in algebra. The National Mathematics Advisory Panel (2008) proposed three curricular strands deemed most essential for students to master prior to receiving algebra instruction. These strands include proficiency with both whole numbers and fractions and specific skills in geometry and measurement. The panel defined proficiency as the ability to “understand key concepts, achieve automaticity as appropriate, develop flexible, accurate, and automatic execution of the standard algorithms, and use these competencies to solve problems” (2008, p. xvii). Likewise, Driscoll (2010) suggested that students’ lack of number sense contributes to difficulties in understanding algebraic structure.

Herscovics and Linchevski (1994) indicated that “the inability to operate spontaneously with or on the unknown indicates the existence of a *cognitive gap*” (p. 63) between arithmetic and algebra, although they also cautioned that this is not the only gap. Young algebra students also tend to have a weak or insufficient understanding of equality (Asquith, Stephens, Knuth, & Alibali, 2007). The ability of students to translate words into mathematical symbols is also an area of concern (Capraro & Joffrion, 2006). However, the research of Nathan and Koedinger

(2000) concluded that, contrary to teachers' beliefs, the ability to reason algebraically actually develops prior to the ability to use algebraic symbolism. This is confirmed through a study of the history of mathematics, as ancient civilizations demonstrated these algebraic thought patterns long before al-Khwarizmi wrote his treatise on algebra in the ninth century (Katz, 2009).

Clearly, certain aspects of algebra are accessible to students prior to a formal algebra course, as indicated by the National Council of Teachers of Mathematics (NCTM, 2000) and in the creation of the Common Core State Standards Initiative (2011). However, for meaningful learning to occur, it is critical for educators to understand students' cognitive progression from thinking arithmetically to reasoning algebraically. Thus, a team of educators at the University of Colorado collaborated on a project to produce research "Supporting the Transition from Arithmetic to Algebraic Reasoning" (STAAR), developing a framework for research in this area, seeking to examine teachers' beliefs, and evaluating best practices regarding algebra education (Nathan & Koellner, 2007).

## Purpose of the Study

The purpose of this study was to determine if mastery of specific eighth-grade standards, as measured by the Indiana Statewide Testing for Educational Progress Plus (ISTEP+) (IDOE, 2011a), can predict success on the Algebra ECA. With this knowledge, curricular decisions can be made to improve student achievement in algebra. Currently, the eighth-grade ISTEP+ results have very little influence over the decisions made in the high school curriculum. What if eighth-grade ISTEP+ scores could be used to drive instruction in algebra? What if algebra instruction could be enhanced by an analysis of the eighth-grade test?

## Significance of the Study

The results of this study have potential curricular and pedagogical value in middle school and high school mathematics classrooms. If middle school teachers have a better understanding as to which specific state standards and skills are prerequisites for success in algebra, they can emphasize those topics in middle school. Professional development can be implemented to ensure that those important concepts are being taught effectively. Likewise, high school teachers can successfully bridge the gap in their instruction if they know which middle school standards are most critical to success in algebra. According to the final report published by the National Mathematics Advisory Panel (2008), algebra teachers should not make assumptions that their students comprehend even basic concepts, such as equality. If mathematics educators stop making assumptions about what students know or should know and rather use the data to drive their instruction, the impact on student achievement could be significant.

## Research Questions

Numerous studies have been conducted to determine predictors of success in algebra, often focusing on demographic variables such as gender, race, and socioeconomic status or certain affective variables such as perceived usefulness and self-confidence (Hahn, 2009; McCoy, 2005). Although these studies may have important implications regarding the accessibility of algebra for all students, the focus tends to be on variables that educators and students cannot control rather than the prior knowledge that might ensure students' success in algebra.

Educators can, however, exercise some control over the mathematics curriculum. Flexer (1984) found that scores on both an algebra prognosis test and a problem-solving test were predictors of success in eighth-grade algebra, but the study does not indicate specifically which cognitive skills students must have to be successful. In this quantitative study, I sought to understand the relationship between the mastery of the Indiana eighth-grade mathematics standards, as measured by the ISTEP+, and success in Algebra I, as measured by the Algebra ECA. Using simple and multiple regression analyses, I sought answers to the following questions:

1. Does the eighth-grade ISTEP+ mathematics composite score predict a significant proportion of the variance in the Algebra ECA score?
2. Do the scores on the seven individual reporting subcategories (i.e. number sense, computation, algebra and functions, geometry, measurement, data analysis and probability, and problem solving) tested on the ISTEP+ predict a significant proportion of the variance in the Algebra ECA score? If a significant proportion of the variance is explained, which of the seven subcategories contributes to this?
3. Does the eighth-grade language arts ISTEP+ score predict a significant proportion of the variance in the Algebra ECA score?

## Limitations and Delimitations

The results of the eighth-grade exam have little to no effect on students. The current policy of IDOE precludes students from being placed in a pre-algebra course in high school, regardless of past performance. The lowest math course that students can take in high school is Algebra I (IDOE, 2013a). However, the Algebra ECA is a high-stakes test that students must pass to earn a high school diploma. Consequently, students may approach the Algebra ECA with a greater sense of urgency. It was assumed, for the purposes of this study, that students gave their best effort on both tests. Additionally, it was assumed that the exams were administrated in an ethical manner, and that both tests yield valid and reliable scores.

The study was conducted using data from a large, suburban school district in central Indiana. The school district is ethnically diverse: approximately 46% Black, 37% White, 9% Hispanic, 6% Multiracial, 1% Asian, and < 1% American Indian and Pacific Islander (IDOE, 2014a). Because the demographic makeup of this school corporation does not reflect the state at large, the results of this study may not be generalizable. The participants in the study were divided into two cohorts; one consisted of students who entered ninth grade in the fall of 2010 and the other consisted of students who entered ninth grade in the fall of 2011. Thus, eighth-grade ISTEP+ data from 2010 and 2011 were compared to Algebra ECA data from 2010 through 2012.

## Definition of Terms

For the purposes of this study, terms were defined as follows:

1. *Algebra.* Although many definitions are possible, algebra is defined as the collection of standards published by IDOE and measured on the Algebra ECA. These standards include the ability to solve and graph both linear and quadratic equations and inequalities, interpret linear and non-linear graphs, solve systems of linear equations and inequalities, and perform operations with polynomials (IDOE, 2012b).
2. *ISTEP+.* Indiana Statewide Testing for Educational Progress Plus program is the standardized testing program mandated by the state of Indiana to assess student achievement in various subjects. The eighth-grade ISTEP+ in mathematics is the most significant assessment in this study; however the ISTEP+ language arts score will also be given consideration in the third regression model.
3. *Number sense.* This is the first subcategory on the ISTEP+ mathematics test. According to *ISTEP+: Grade 8 Mathematics Blueprint* (IDOE, 2012c), this subcategory is assessed

with questions involving rational and irrational numbers, using and understanding the laws of exponents, and understanding the relationship between squaring a number and its square root.

4. *Computation* At the eighth-grade level, students are assessed on their ability to add, subtract, multiply, and divide rational numbers as the second subcategory on the ISTEP+ (IDOE, 2012c).
5. *Algebra and functions.* This is the third subcategory on the ISTEP+. Although it does not encompass all of the standards tested on the Algebra ECA, it includes solving simple equations and inequalities, systems of equations, and using tables, equations, verbal expressions, and graphs to interpret linear and quadratic equations (IDOE, 2012c).
6. *Geometry.* As measured by the ISTEP+, the fourth subcategory of geometry includes understanding the properties of geometric shapes, identifying basic geometric transformations, and using the Pythagorean theorem (IDOE, 2012c).
7. *Measurement.* Measurement includes the ability to convert between measurements, use ratio and proportion as related to volume and area, and find perimeter, area, volume, and surface area of regular and irregular shapes. This is the fifth subcategory on the ISTEP+ (IDOE, 2012c).
8. *Data analysis and probability.* As the sixth subcategory on the ISTEP+, data analysis requires the ability to interpret and analyze graphical representations and use statistical measures and the basic counting principle (IDOE, 2012c).
9. *Problem solving.* Problem solving involves the ability to demonstrate flexibility in the approach to problems as demonstrated by multiple approaches to problems, justifying

solutions, and identifying relevant information (IDOE, 2012c). This is the seventh subcategory on the ISTEP+.

10. *Algebra ECA.* The Algebra End-of-Course Assessment is the standardized test given at the completion of Algebra I in Indiana. Students are required to earn a minimum passing score to graduate from high school.
11. *Success in algebra.* Success was defined as earning the minimum passing score on the ECA, as defined by IDOE.

**Summary**

This quantitative study determined if the eighth-grade ISTEP+ mathematics test predicted a significant proportion of the variance on the Algebra ECA. Perhaps even more important, it attempted to determine if specific prior knowledge, as measured by the seven reporting subcategories on the ISTEP+, could be used to predict success in algebra. Additionally, the study determined if success on the eighth-grade ISTEP+ language arts test could be used to predict a significant proportion of the variance on the Algebra ECA. Chapter 2 will explore the literature relevant to this study.

# CHAPTER 2

## REVIEW OF LITERATURE

### A Brief History of Algebra in the School Curriculum

Algebra has long been an integral part of the mathematics curriculum in America. Prior to the nineteenth century, algebra was reserved for university students, but by 1820 Harvard University began requiring it for admission and other universities soon followed (Kilpatrick & Izsák, 2008). As high school enrollments experienced unprecedented growth from 1890 when only 7% of the eligible population attended high school to 1940 when almost 75% of the U.S. population was enrolled, the percentage of students taking algebra actually decreased (Jones & Coxford, 1970). Even for much of the twentieth century, algebra was deemed necessary only for those students who might attend college, while in the name of social efficiency, arithmetic was essential for everyone (Kliebard, 1995).

However, Russia's launch of Sputnik in 1957, seen as a challenge to national security, vaulted American mathematics education to the forefront of curriculum reform (Schoenfeld, 2004). Mathematics and the sciences suddenly seemed to have an inherent function, rather than being a mere exercise of the mind, and the "New Math" era emerged. In the study of algebra, New Math meant a shift from symbol manipulation and solving simple equations to a study of functions, patterns, inequalities, and deductive reasoning, traditionally reserved for geometry (Kilpatrick & Izsák, 2008). However, in reaction to the New Math movement, there was a public outcry in the early 1970s to return to the basics (Schoenfeld, 2004). Thus, the massive pendulum of curriculum reform swung back, focusing once again on symbol manipulation and other procedural skills.

As another decade passed, mathematics education and the back-to-basics movement was viewed as a dismal failure. Students were still failing algebra in alarming numbers (Kilpatrick & Izsák, 2008), and most students lacked problem-solving skills (Schoenfeld, 2004). In light of the economic crisis of the early 1980s, the National Commission on Excellence in Education released *A Nation at Risk*: *The Imperative for Educational Reform* in 1983, lamenting the state of American education and demanding significant reforms (Alters, 2008). The release of this report, coupled with lagging U.S. scores on the Second International Mathematics Study, spurred mathematics educators to seek genuine algebra reform (Schoenfeld, 2004).

## The Algebra Reform Movement

The call for algebra reform was based on two different viewpoints. First, there was a perception that the state of mathematics education was in disrepair. American students were not able to compete with their international counterparts in the areas of math and science, which was seen as a threat to national security and America's position in the global economy. The back-to-basics movement of the previous decade had left students without basic problem-solving skills or the ability to reason mathematically (Schoenfeld, 2004). As a result, NCTM (1980) released *An Agenda for Action*, calling for a focus on problem solving within the context of mathematical content.

The second guiding principle of algebra reform was the idea that algebra should not be reserved merely for those students deemed college-bound; rather, algebra education should be for everyone, regardless of ethnicity or socioeconomic status. Taking this position a step further, leading civil rights activist Robert Moses challenged the conventional belief that algebra success depended on an innate ability and posited the idea that, given the proper context, all children can learn algebra (Moses & Cobb, 2001). Moses developed The Algebra Project, used with inner-city middle school students, successfully providing early algebra education to those who had traditionally been denied access. As Silva, Moses, Rivers, and Johnson (1990) wrote, "Mathematics and science proficiencies are taking their place alongside reading and writing as requirements of citizenship. Those without these tools are fast losing access to the new political and economic institutions" (p. 377). Because women and minorities were severely underrepresented in the areas of math and science, the idea of equal access to algebra was seen as a civil rights issue (Silva et al., 1990).

NCTM (1989) had already stated this sentiment with the release of *Curriculum and Evaluation Standards for Teaching Mathematics* (the *Standards*), along with a call for equal opportunity. As Schoenfeld (2004) wrote, "Whatever the intention may have been, the reality was that the traditional curriculum was a vehicle for social efficiency and the perpetuation of privilege" (p. 268). The *Standards* were more of a vision statement as opposed to a blueprint for curriculum, espousing a constructivist approach to mathematics education. The focus was on process standards with a call for new teaching methods that helped students engage in mathematical sense making through activities such as group work and project work, as opposed to worksheets or textbook drills, and discussion as opposed to lecture. Pedagogically foreign to the traditionalist viewpoint and seen as a challenge to the social order, the *Standards*, and much of the resulting curricula, were met with resistance from anti-reform groups (Schoenfeld, 2004).

## Algebra Today

Reform can be a slow, tedious process, and mathematics education continues to evolve, hopefully for the betterment of our youth. As a revision of the *Standards*, NCTM (2000) released *Principles and Standards for School Mathematics*, which called for the infusion of algebraic concepts in mathematics curricula at the elementary level. As NCTM offered a vision for mathematics curricula, many states quickly followed with their own more specific versions of standards. However, there was no real consistency from state to state, leading to a controversial call for a national curriculum.

Through an initiative started by the National Governors Association for Best Practices in conjunction with the Council of Chief State School Officers, the *Common Core State Standards* were developed and subsequently adopted by 45 states as of 2012 (Common Core State Standards Initiative, 2011). Based on research, the *Common Core* is a set of rigorous standards intended to prepare all students for college or career. These standards focus on conceptual knowledge in addition to procedural knowledge, with a call for fluency in computation. Like the *Principles and Standards* that preceded the *Common Core*, algebra is a strand of the curriculum at each grade level, although it has varying levels of emphasis. Thus, the current trend is to introduce algebra or algebraic thinking at earlier ages, helping students make the transition from arithmetic to algebra. This movement agrees with the recommendation of Schoenfeld (1995) that students should have significant exposure to algebraic concepts before they take a formal class.

## Algebra in the Early Grades

Why should students study algebra at younger ages? What form should this introduction to algebra take and are students developmentally ready for such exposure? According to Cai and Moyer (2008), introducing younger children to algebraic concepts does not mean forcing formal algebraic study on young minds, but rather changing the way arithmetic is taught. "Resistance to algebra in middle and high school would be reduced if we could remove the misconceptions that arithmetic and algebra are disjoint subjects" (Cai and Moyer, 2008, p. 170). If algebra and arithmetic are taught in conjunction with one another, students can see the relationship between the two branches of mathematics, and algebra will seem less intimidating (Cai & Moyer, 2008).

Cai and Moyer (2008) analyzed curricula in China and Singapore to determine how children are exposed to algebraic concepts in those countries. They discovered three common practices in the curriculum that aid in developing algebraic thought. First, students are introduced to addition and subtraction simultaneously in first grade. Subtraction is first taught as the reverse of addition, reinforcing the "doing-undoing" concept needed to solve algebraic equations. This same practice is used in second grade when students are taught multiplication and division at the same time.

Second, the use of pictures to model problems is common as early as first and second grade. As students reach fourth and fifth grades they are routinely solving more complicated problems, illustrated with rectangles, without using symbolic algebraic equations. Finally, the third common practice in this curriculum is to solve problems with different methods, using both arithmetic and algebraic approaches. Often teachers will model different approaches for students, demonstrating that problems can be solved with multiple approaches. This practice helps students develop a flexibility of thought that is important for the development of algebra (Cai & Moyer, 2008).

Also giving credence to early algebraic thought, the research of Nathan and Koedinger (2000) concluded that, contrary to teachers' beliefs, the ability to reason algebraically actually develops prior to the ability to use algebraic symbolism. In this experimental study, teachers were given both “start unknown” and “result unknown” problems in story form, word equation, and symbolic notation and were asked to rank them by degree of difficulty for their students. Teachers largely thought the symbolic problems would be the easiest for students to solve and story problems would present the most difficulty. However, when the same problems were given to students to solve, the word equations or story forms were solved correctly with greater frequency (Nathan & Koedinger, 2000). These results might indicate that context is critical when learning algebra and it has implications for the way teachers should choose to introduce equation solving to their students.

## Algebra Readiness

What does make a student ready to study algebra? The authors of *Implementing the Common Core State Standards* (Common Core State Standards Initiative, 2011) claimed that mastery of the K–7 standards will prepare students for the formal study of algebra. However, many students are still failing their first formal algebra class. So, what do they need to know to experience success? Though not necessarily research based, the National Mathematics Advisory Panel (2008) proposed three clusters of knowledge to be most critical to success in algebra: (a) understanding of whole numbers, (b) understanding of fractions, and (c) understanding certain aspects of geometry and measurement. Thus, the Panel suggested a solid link between arithmetic and algebra.

### Proficiency with Fractions

Driscoll’s (1982) *Research Within Reach: Secondary School Mathematics* indicated that secondary teachers regularly reported concerns over their students’ inabilities to understand fractions and provide reasonable estimations, demonstrating an overall lack of number sense. There is a common perception among researchers and educators alike that fluency with fractions is an important component of algebraic thinking (Driscoll, 1982; National Mathematics Advisory Panel, 2008; Wu, 2001). Wu (2001) wrote, “The proper study of fractions provides a ramp that leads gently from arithmetic up to algebra. But when the approach to fractions is defective, that ramp collapses, and students are required to scale the wall of algebra not at a gentle slope but at a ninety degree angle” (p. 17).

Brown and Quinn (2007) sought to establish the link between fraction proficiency and success in algebra through an ex post facto design study comparing students’ grades on exams in elementary algebra and intermediate algebra with their grades on a fraction competency test. Using a relatively small sample of 191 students at one high school, they determined that there was a statistically significant relationship between the scores on the fraction competency test and success in algebra ($p < 0.05$) for both elementary algebra students and intermediate algebra students. This conclusion is not surprising, because the underlying structure of algebra is rooted in an understanding of arithmetic structure (Brown & Quinn, 2007).

Using regression analyses, Siegler et al. (2012) found that both an understanding of fractions and whole-number division were significant predictors of success in high school mathematics, even when statistically controlling for general intellectual ability and other demographic factors. Using longitudinal data from two different populations in both the United States and the United Kingdom yielded the same results, leading to the generalizability of the study. According to these researchers, these two skills are necessary for solving many advanced mathematics problems. Yet students fail to master these skills at a high rate, leading to the high degree of predictability. One reason cited for students failing to master these skills was poor instruction in these topics (Siegler et al., 2012).

As Siegler et al. (2012) focused primarily on operations with fractions, Booth (2012) sought to understand how students' understanding of fraction and whole number magnitude might be related to algebra readiness. Students were given three different tasks to be completed using number lines of various scales; two involved whole number magnitudes and one involved fractional magnitudes. Students were also given several algebra readiness tasks. Booth (2012) concluded that fractional magnitude knowledge had the greatest impact on students' algebra readiness, perhaps because it was indicative of a deeper number sense.

**Working with the Unknown**

Herscovics and Linchevski (1994) also conducted research on the transition from arithmetic to algebra, proposing "the inability to operate spontaneously with or on the unknown indicates the existence of a cognitive gap that can be considered a demarcation between arithmetic and algebra" (p. 63). To confirm their hypothesis, they designed a two-part experiment where the first part investigated students' basic arithmetic skills deemed necessary for equation solving and the second part assessed the types of equations that seventh graders could solve without prior instruction. Using a limited sample of 22 students in a parochial school, the researchers conducted individual interviews with the students. The first part of the experiment that assessed arithmetic skills consisted of a series of questions designed to determine the students' understanding of equality, order of operations, cancellation (+5-5), and concatenation. All of the students seemed to have a solid grasp of the arithmetic assessed in the interview.

The second part of the experiment dealt with the students' ability to solve equations without prior instruction (Herscovics & Linchevski, 1994). Restricting the equations to natural numbers, the researchers presented equations involving all four operations. Additionally, they varied the placement of the variable, the size of the numbers, the number of operations required, and the direction of the equation. Of special significance to the researchers were examples of when the unknown occurred twice, whether on the same side of the equal sign or on opposite sides. Although the students were overwhelmingly able to solve all of the equations presented, their method of solving indicated a numerical approach, such as substitution, indicating an inability to work with the unknown. Another interesting observation was the students' misunderstanding of the minus sign, where they would detach it from the following number. For example, in a problem that contained x – 2 + 3, students would try to add the 2 and the 3 first. This may be another indication of a gap between arithmetic and algebra (Herscovics & Linchevski, 1994).

Another aspect of the unknown is the multiple conceptions of variable that are used in algebra. Often students will say that they understood math until there were more letters than numbers in the problems. These students are citing one of the fundamental reasons that students have difficulty learning algebra: they have a problem making sense of the symbolism involved. Although students can frequently grasp that the symbol *x* can take on different values, the difficulty lies in the fact that the concept of variable has plural meanings. According to Malisani and Spagnolo (2009), the concept of variable can refer to a specific, yet unknown, number whose value can be calculated by a specific procedure or a series of symbol manipulations as you might find in the equation 3x + 2 = 11. This view of the variable seems the most natural to students. However, variables can also be used to generalize arithmetic, such as in the equation a(b + c) = ab + ac, an illustration of the distributive property. In this case the symbols *a*, *b*, and *c* do not have specific number values; experienced mathematicians understand that these symbols could refer to any number. A third concept of the variable is that of functional relationship, meaning that the letter could take on a range or set of values, such as in $y = 3x + 2$.

Through their research, Malisani and Spagnolo (2009) wanted to determine which concept of variable students would use and if the different conceptions would interfere with each other, perhaps limiting student responses. They also wanted to observe the process by which students translated from natural language to algebraic language. Although they had a limited sample of 111 Italian students, Malisani and Spagnolo concluded that their subjects predominantly used natural language as opposed to symbolic language to solve the problems. Additionally, they overwhelmingly viewed the variable as a single unknown, rather than as a function (Malisani & Spagnolo, 2009). Thus, research indicates that it is important for teachers to develop the concept of variable methodically and explicitly, without assuming that students will recognize the various conceptions.

In another study investigating students' understanding of variables, Christou and Vosniadou (2012) suggested that students bring to their understanding of variables their understanding of the concept of number, which leans heavily toward a natural number prejudice. Using a series of three experiments, they confirmed their hypothesis. Students overwhelmingly chose only natural numbers for variables in both open-ended and multiple-choice questions. These results have implications for other areas of algebra such as linear functions. If students have a tendency to only consider natural numbers, they may interpret lines as a set of discrete points, failing to understand the concept of continuity. As these researchers concluded, "developing the mathematical concept of real variable may require understanding the concept of rational number in arithmetic, something few students have achieved by the time they are exposed to variables" (Christou & Vosniadou, 2012, p. 21).

**Understanding the Equal Sign**

When transitioning from arithmetic to algebra, students must also increase their understanding of the equal sign. Frequently adults will define *equal* to mean *the same as* but an even greater algebraic understanding would be that of an *equivalence relation*. However, many elementary, middle school, and even high school students do not understand the equal sign in this way. Behr (1976) and researchers at Florida State University and the Tallahassee Project for the Mathematical Development of Children conducted interviews with students in Grades 1, 2, 3, and 6 to determine their concept of the equal sign. He concluded that students overwhelmingly see the equal sign as a “do something signal” rather than as a relational symbol. Additionally, these students exhibited rigidity in their thinking, indicating that equations had to have a specific structure, such as the answer always coming after the equal sign (Behr, 1976). Baroudi (2006) posited that the use of the calculator may also contribute to this notion of equality.

In a more recent study, Asquith et al. (2007) considered teachers’ understanding of student thinking regarding the equal sign. Teachers selected for this study were asked to identify the methods students would use to solve certain problems and provide a list of the prior knowledge students would bring to the problem. The research showed that some teachers recognized that students frequently viewed the equal sign as an operator, but they misjudged the extent to which this mistake was made by students. Upon further questioning, many teachers thought that the students’ vast exposure to the equal sign would indicate a fuller understanding of it as relational (Asquith et al., 2007). From this research, it would appear that middle school teachers, who are often content-area specialists, are making assumptions about how elementary teachers are instructing regarding the equal sign. Because of this tendency of students to look at the equal sign as an operator, middle and high school teachers must alter their teaching strategies, stop assuming that students understand the relational aspect of the equal sign, and explicitly address the issue. Additionally, elementary school teachers may need additional professional development so they can learn methods that would reinforce the idea of equivalency in their instruction, changing the way arithmetic is taught and thus allowing students to form a solid conceptual base before they even attempt algebra.

## Standardized Testing

Since the inception of public schools, students have been required to participate in various assessments, all with different consequences. Some assessed aptitude as others assessed achievement, but few were attached to the high stakes currently seen in education today. In response to Russia's launch of Sputnik, an act seen as a triumph over the achievement of American scientists, Congress adopted the Elementary and Secondary Education Act of 1965 (ESEA), marking the first time standardized assessment data were utilized in educational polices and decision making (Nichols & Berliner, 2007). Concerns regarding American education seemed to grow from this time, culminating in the release of *A Nation at Risk* report, prepared by the National Commission on Excellence in Education, which served as an impetus for wide-sweeping reform and made recommendations such as lengthening the school year and requiring a more rigorous curriculum, especially at the secondary level (Alters, 2008).

In 2002, Congress enacted No Child Left Behind Act of 2001 (NCLB), a major overhaul of the ESEA, requiring states to assess students in mathematics and science in Grades 3–8 and once in high school. These data were then used as an accountability measure, determining whether schools were making adequate yearly progress (AYP). With this new law, schools failing to meet AYP faced dire consequences, including decreased funding, reassignment or firing of faculty, or simple closure. Students who failed also faced consequences such as remediation or possibly retention (Caffrey, 2010). NCLB marked unprecedented accountability for educators and served as a harbinger of true high-stakes testing. However, it is important to note that the use of high-stakes testing is much maligned by many leading educators and the effectiveness to increase achievement is often debated. Nichols and Berliner (2007) considered such testing a corruption of schools, citing increased evidence of cheating by teachers and administrators and unethical practices that limit the accessibility of education for all students.

## History of Standardized Testing in Indiana

The history of standardized testing in Indiana has paralleled the changes in national legislation. Prior to the publication of *A Nation at Risk*, local governments maintained primary control of education in their communities. In 1977, through separate proposals from the Indiana General Assembly and the State Board of Education, schools were required to identify minimum basic skills, test students at least three times before tenth grade, and administer an additional graduation test in eleventh grade. Schools had control over exactly what was tested, when it was tested, and who was tested (Bull, 1998).

In response to *A Nation at Risk*, the Indiana General Assembly passed a bill in 1984 requiring the administration of the Indiana Basic Competency Skills Testing and Remediation program (IBCST) in Grades 3, 6, and 8. This is the first time a state-mandated standardized test was administered in the state, effectively wrenching control of curriculum and assessment away from local districts. The A+ Program legislation, passed in 1987, changed the name of the test to the Indiana Statewide Testing for Educational Progress (ISTEP) and added testing in additional grade levels. The law linked school funding to test scores, and required the remediation and possible retention of students who did not pass, thereby holding schools and students accountable for the results (Bull, 1998).

Further legislative changes occurred in 1995 when the ISTEP morphed into the ISTEP+, a criterion-referenced and norm-referenced test with both multiple choice and applied skills questions. This test was given in the fall to students in Grades 3, 6, 8, and 10, with the passage of the high school test becoming a requirement for graduation. After adoption of the Indiana Academic Standards in 2000, the ISTEP+ became a criterion-referenced test only, aligned to the Academic Standards. In 2003, to comply with the requirements outlined in NCLB, the ISTEP+ began testing students in Grades 3–8 and 10. After much public and political pressure, the ISTEP+ test was moved from the fall to the spring in 2008, in an effort to hold teachers more accountable for the results. Additionally, in the following year, a new graduation qualifying requirement was enacted: students had to demonstrate mastery on the standards assessed by ECA exams in Algebra I and English 10 (IDOE, 2011a).

## Studies Linking Achievement to Test Scores

Few studies have attempted to link results on one standardized test to achievement on a second standardized test. However, Hull and Tache (1993) correlated the scores of eighth-grade students on the Iowa Test of Basic Skills (ITBS) to their scores the following year on Ohio's Ninth-Grade Proficiency Test over a period of three years. They determined there was a significant correlation between the scores on the two tests, the strongest being in the mathematics and citizenship scores. According to the results of their study, students who scored below average on a particular section of the ITBS also tended to fail the corresponding section on Ohio's Proficiency Test. Although the generalizability of their study may be limited because their subjects were restricted to one Ohio community, Hull and Tache concluded that scores on the ITBS could be used to predict those who may benefit from intervention programs prior to taking the Ohio Ninth-Grade Proficiency Test (Hull & Tache, 1993). However, the design of this study did not allow the researchers to determine which specific mathematics skills the students might be lacking.

Konstantopoulos, Miller, van der Ploeg, Li, and Traynor (2011) studied the effects of Indiana's formative assessments or benchmark tests on mathematics achievement. Indiana uses two different commercially developed, computer-delivered formative assessments that are aligned to the Indiana standards. Students in K–2 are assessed using *mCLASS:Math*, developed by Wireless Generation, and students in Grades 3–8 are assessed using CTB/McGraw-Hill's *Acuity* tests. The purpose of these assessments, given three times throughout the school year, is to provide teachers with individualized data for students so they can adjust instruction.

Through a randomized theoretical design, Konstantopoulos et al. (2011) determined that the overall treatment effect for Grades K–8 was positive, though not statistically significant. However, when limiting the analysis to Grades 3–8, the results were statistically significant ($p < 0.05$). These results may be a reflection of the two different assessments used, *mCLASS:Math* versus *Acuity*. When the individual grade levels were analyzed, the largest treatment effect occurred in fifth and sixth grades (Konstantopoulos et al., 2011). It would seem that the assessments themselves could not improve mathematics achievement, but rather it is the individual classroom teacher's use of the data that might improve achievement. A follow-up study should be conducted determining how teachers actually used the information obtained with the assessment. Also an indication of the type of training teachers received on administering, interpreting, and utilizing the results could provide helpful information regarding the effectiveness of these benchmark tests as well.

A third study, by Kriegler and Lee (2007), looked at the effects of using standardized test data to place students into eighth-grade algebra. The California Mathematics Framework indicates that Algebra I should be the grade-level content for all eighth-grade students, although the decision to place a student in algebra could be decided by school and district policy. These researchers considered the scores students earned on the seventh-grade and eighth-grade California Standards Tests (CST) and the Algebra CST. They also took into account the types of placement policies the schools used. Schools that placed only students who scored *proficient* on the seventh-grade test were considered *conservative* whereas schools that did not were considered to have an *aggressive* placement strategy. The results indicated that proficiency on the seventh-grade test was an important indicator for success in eighth-grade algebra (Kriegler & Lee, 2007). However, this study does not reveal the specific mathematics skills that might predict success in algebra.

In a study similar to the one I conducted for this dissertation, Liang (2009) studied the relationship between students' scores on two consecutive years of testing data on the CST. Using data from the California statewide data base, she conducted a simple regression analysis using the score on the seventh-grade CST as the predictor variable and the score on the Algebra CST given in eighth grade as the criterion variable. Liang determined that the seventh-grade score was a reliable predictor of proficiency on the eighth-grade Algebra CST, accounting for 61% of the variance. Additionally, in an attempt to determine if specific prior knowledge affected the algebra scores, a multiple regression analysis was conducted to determine the relationship between the six individual reporting categories and the score on the Algebra CST. It was determined that the reporting category of rational numbers had the largest effect, contributing to 47% of the variance, with the other categories having a minimal effect on scores. However, it is unknown what skills this category of rational numbers might have assessed. Furthermore, a principal factor analysis determined the seventh-grade test was truly one-dimensional, with one latent factor underlying the individual reporting categories. This conclusion begs further study to determine what that factor is and how it might truly affect achievement in algebra (Liang, 2009).

## Summary

Algebra has had a pivotal role in mathematics education throughout history. It is seen as a gateway to all future success in math and science as well as a civil rights issue. Educational policies must allow students early access to algebra, and educators must figure out ways to help these students be successful. As schools are increasingly held accountable for students' algebra proficiency, as measured by standardized tests, it is imperative that educators be able to identify those skills that students must have to succeed in algebra.

Research clearly shows that students must make a transition from arithmetic thought to algebraic reasoning. Several areas of concern as students make this transition include their ability to work with fractions, their understanding of the equal sign, and their understanding of the concept of variable. Also, students' ability to transition from verbal language to the symbolic language of algebra is critical. However, few studies really indicate the exact skills students must bring with them into an algebra class.

This study sought to determine which skills measured by the eighth-grade ISTEP+ contribute to success in Algebra I, as measured by the Algebra ECA. If algebra success can be attributed to the mastery of specific standards, educators would be able to focus instructional time and resources on those skills deemed most important. In Chapter 3, I discuss the methodology that was employed to determine if there is a relationship between specific eighth-grade mathematics standards measured on the ISTEP+ and success on the Algebra ECA.

# CHAPTER 3

## METHODOLOGY

The importance of algebra in the high school curriculum is well documented, yet the subject continues to prove difficult for students. The literature review revealed several factors that might attribute to this. Herscovics and Linchevski (1994) concluded that students experienced a cognitive gap as they transitioned from arithmetic to algebra. Other research indicated that a lack of number sense, an inability to work with fractions, and misunderstanding algebraic symbolism all contributed to student difficulties in algebra (Asquith et al., 2007; Brown & Quinn, 2007; Capraro & Joffrien, 2006). This study sought to determine the prior knowledge that students need in order to succeed in algebra, thereby increasing mathematical success at subsequent levels of study.

Using quantitative methods, this study determined if students' scores on the eighth-grade mathematics test could be used to predict a significant proportion of the variance on the Algebra ECA. More specifically, I sought to determine if there are any particular individual eighth-grade standards that were more significant than others in predicting the variance of scores on the Algebra ECA. If educators can gain a thorough understanding of the skills and concepts students need to develop in order to experience success in algebra, changes can be made in both middle school and high school curricula and pedagogy. The results of this study can also be used to develop remediation or intervention strategies.

The research methodology utilized in this study is detailed in this chapter. First, the research questions and the null hypotheses are stated. The subsequent sections describe the population chosen for the study and the methods of data collection and analysis. A description of both the ISTEP+ mathematics test for eighth grade and the Algebra ECA is provided, including a brief discussion of the test construction.

## Research Questions

Answers to the following questions were sought in the context of this study:

1. Does the eighth-grade ISTEP+ mathematics composite score predict a significant proportion of the variance in the Algebra ECA score?
2. Do the scores on the seven individual reporting subcategories (i.e. number sense, computation, algebra and functions, geometry, measurement, data analysis and probability, and problem solving) tested on the ISTEP+ predict a significant proportion of the variance in the Algebra ECA score? If a significant proportion of the variance is explained, which of the seven subcategories contributes to this?
3. Does the eighth-grade ISTEP+ language arts score predict a significant proportion of the variance in the Algebra ECA score?

## Null Hypotheses

$H_o1$: The eighth-grade ISTEP+ mathematics composite score does not predict a significant proportion of the variance on the Algebra ECA.

$H_o2$: The scores on the seven individual reporting subcategories tested on the ISTEP+ do not predict a significant proportion of the variance on the Algebra ECA.

$H_o3$: The score on the eighth-grade ISTEP+ language arts test does not predict a significant proportion of the variance on the Algebra ECA.

## Population

This study took place in a large, suburban school district in a metropolitan area of Indiana, with a total enrollment of 11,741 students in the 2011 school year. Ethnically diverse, the district is 46% Black, 37% White, 9% Hispanic, 6% Multiracial, 1% Asian, and less than 1% American Indian and Pacific Islander according to data supplied by IDOE (2014a). A total of 16% of the students receive special education services, and 6% are classified as English Language Learners (IDOE, 2014b). Additionally, this is a Title I school district and 65% of the students receive free or reduced-cost lunch. The graduation rate for the district was approximately 88% in 2011 and the attendance rate was reported at 97% (IDOE, 2014a).

The sample consisted of two separate cohorts of students; the first cohort included those students entering ninth grade in 2010 and the other included students entering ninth grade in 2011. Each cohort was further divided into two groups. One group of students, referred to as the *advanced cohort* took both the eighth-grade ISTEP+ and the Algebra ECA in the same year. These students were considered advanced because they completed algebra in eighth grade. The second group, referred to as the *average cohort*, took the ISTEP+ in eighth grade and the Algebra ECA upon completion of algebra in ninth grade. Thus, for the purposes of this study, ISTEP+ scores from 2010 and 2011 were used along with Algebra ECA scores from 2010 through 2012.

Because students are sometimes absent during testing, a list-wise deletion of missing data was conducted resulting in any student who did not have both an ISTEP+ score and an Algebra ECA score being removed from the analysis. Additionally, students who did not have a score in every reporting subcategory, indicating they were absent for a portion of the test, were also removed from the analysis. The exact number of participants varied for each cohort, ranging from a population size of 264 to 454 participants. According to Field (2009), this sample size would be more than adequate for seven different predictor variables where a medium effect is expected. The students attended one of three middle schools in the district, but they all attended the same high school. The following sections will describe both tests and their construction in detail.

## Eighth-Grade ISTEP+ for Mathematics

Currently, the ISTEP+, a criterion-referenced test, is administered to all eighth-grade students in Indiana. The assessment, developed independently by CTB/McGraw-Hill, is aligned to the Indiana Academic Standards, and its purpose is to measure student achievement in eighth-grade mathematics. The test, administered in the spring, is given in two parts. An applied skills test is given first in March, and a multiple choice test is administered at the beginning of May. Students may use calculators on parts of the mathematics assessment (IDOE, 2011b).

The ISTEP+ measures student achievement on the seven Indiana Academic Standards and individual scores on each standard are reported. Based on the score earned, students are assigned to one of three performance categories: pass+, pass, or did not pass. According to the *ISTEP+: Grade 8 Testing Blueprint* (IDOE, 2012c), the standards and their approximate weights are as follows:

1. Number sense (8%–18%). This standard includes questions on scientific notation, rational and irrational numbers, exponents, and square roots.
2. Computation (5%–15%). This standard may include questions regarding adding, subtracting, multiplying, and dividing with rational numbers and calculating simple interest.

3. Algebra and functions (15%–25%). The largest section of the test, this standard may include questions on writing and solving equations and inequalities, systems of equations, evaluating and simplifying expressions, understanding multiple representations with linear and quadratic functions.
4. Geometry (5%–15%). This section includes questions regarding the basic properties of geometric shapes, identifying transformations, and using the Pythagorean theorem.
5. Measurement (9%–19%). An extension of geometry, this section may include questions regarding measurement conversion, proportional figures, and calculating area, volume, and perimeter of regular and irregular shapes.
6. Data analysis and probability (10%–20%). This section may include questions involving organizing and interpreting data in graphs, determining the reasonableness of certain conclusions, using statistical measures, probability and the basic counting principle.
7. Problem solving (13%–23%). This standard is interwoven with the other standards. Questions include identifying relevant and irrelevant information, using various strategies to solve problems, and justifying solutions.

**Algebra End-of-Course Assessment**

The Algebra ECA is a criterion-referenced test administered to all Indiana students after the completion of a first-year algebra course, whether that is in middle school or high school. Middle school students who complete algebra take both the ISTEP+ and the Algebra ECA in the same academic year. Developed by Questar Assessment for IDOE, the Algebra ECA consists of four item types: multiple-choice, constructed response, gridded response, and graphing. Students may use their own calculators on a portion of the test. In compliance with NCLB, students are placed into one of three categories based on their score: pass+, pass, and did not pass (IDOE, 2012a).

Aligned to the Indiana Academic Standards for Algebra, the test items are divided into five reporting categories. In addition to the overall score, students receive a score on each of the individual reporting categories. According to the *ISTEP+: Algebra I Graduation Exam Blueprint* (IDOE, 2012b), the reporting categories and their approximate weights are as follows:

1. Solving linear equations and inequalities (15%–25%)
2. Graphing and interpreting linear and non-linear relations (20%–30%)
3. Systems of linear equations and inequalities (15%–25%)
4. Polynomials (15%–25%)
5. Solving and graphing quadratic equations (10%–20%)

## Test Construction

Although both the ISTEP+ and the Algebra ECA were developed by two different testing companies, both were written using the same psychometric approach. Using item response theory (IRT), the questions on each test were analyzed to determine the level of difficulty of each item and the level at which each item accurately identifies students who do or do not have the skill being assessed. Different statistical models within IRT were used for the different types of questions, whether multiple choice or open ended. Pattern scoring, as opposed to raw scoring, is then used to score the tests. The ability scores are placed on a scale. The ISTEP+ scores are scaled on a continuum for testing in Grades 3–8, allowing educators to determine if growth has occurred. However, the Algebra ECA and the ISTEP+ are not on the same scale, nor do they assess the same standard strands. Thus, growth from eighth-grade math to algebra cannot be determined simply by looking at the scores (IDOE, 2011a).

Individual academic standards scores are reported for each individual student, according to the Indiana performance index. More advantageous than a simple percentage, this index is a statistical value that reflects the number of items a student would have answered correctly if he or she had responded to 100 similar items for the specific standard (IDOE, 2011a). Because these individual standards scores are available to both educators and parents, the results of this test do have the opportunity to impact instruction. The individual strengths and weaknesses of each student, and each class, can be analyzed, allowing for the implementation of curricular and pedagogical changes as a result of assessment. The following sections describe how the data from these tests were collected and analyzed.

## Method of Data Collection

I did not have any contact with the subjects in regard to the data collection. Rather, to preserve the anonymity of the subjects, an employee of the school district assigned a dummy testing number to each participant, then the data were disseminated to me via Microsoft Excel spreadsheets. The data included the dummy testing number, the eighth-grade ISTEP+ language arts score, the eighth-grade ISTEP+ mathematics score, the scores from the seven reporting subcategories on the ISTEP+ mathematics exam, and the Algebra ECA score. These data were then uploaded to SPSS, statistical software, in order to conduct the statistical analysis. The data were stored securely on a personal computer, accessible only by me.

## Method of Data Analysis

Simple regression analysis was used to test the first and third null hypotheses, and multiple regression analysis was utilized to test the second null hypothesis. SPSS was used to perform these analyses. An advantage of regression analysis is that it has a predictive quality.

Theoretically, a model can be obtained that could be used to predict the dependent variable. This method of analysis was chosen in an effort to explain the largest amount of variance.

First, a simple regression was conducted to determine if the eighth-grade ISTEP+ mathematics score predicted the Algebra ECA score. However, the standards measured on each test are different. This study sought to understand specifically what prior knowledge students might need to be successful in algebra. Thus, a second multiple regression model was constructed with the seven reporting subcategories of the ISTEP+ as independent variables and the Algebra ECA score as the dependent variable. The intent of this approach was to determine which individual standards have the largest impact on the Algebra ECA score. Developing an understanding of the specific skills on which students need to demonstrate proficiency could help educators as they plan for instruction and revise curricula.

Because other factors could also contribute to the variance, a third model was applied, using the eighth-grade ISTEP+ language arts score as the independent variable and the Algebra ECA as the dependent variable. The language arts score was chosen as a potential variable based on the results of Liang (2009) who also included this variable in her study. Although different standardized tests were used in Liang's study in California, she found that the language arts score was the second leading predictor variable in her multiple regression analysis. Walker (2012) also determined a significant relationship between reading fluency and mathematics constructed response scores on the ISTEP+ exam for third-, fourth-, and fifth-grade students. Thus, because students must be able to read questions and directions on the test, it seems reasonable to conclude that there may be a relationship between the mathematics score and the language arts score. All three models were analyzed for issues of multicollinearity and violation of statistical assumptions.

## Summary

This study utilized data from the eighth-grade mathematics ISTEP+ administered in the springs of 2010 and 2011 along with data from the Algebra ECA administered in the springs of 2010 through 2012 to investigate the relationship between prior academic achievement and success in algebra. The data were collected from one large, diverse school district in central Indiana, using two cohorts of students, one entering ninth grade in 2010 and the other entering ninth grade in 2011. Three regression models were developed using the ISTEP+ mathematics score, the seven reporting subcategory scores, and the ISTEP+ language arts score. Each model was analyzed for goodness of fit, generalizability, and multicollinearity. Additionally, all assumptions were checked. The results of this study will contribute to the existing knowledge base regarding predictors of success in high school algebra. Chapter 4 offers a detailed analysis of the data.

# CHAPTER 4

## ANALYSIS OF RESULTS

This chapter is a discussion of the results of the quantitative analysis conducted to test the three null hypotheses posed in this study. The first section will explore the results of the first simple regression model which sought to determine if the eighth-grade ISTEP+ mathematics test could be used to predict a significant proportion of the variance in the Algebra ECA. Next, the results of the multiple regression model, developed to determine if any of the seven individual reporting subcategories could be used to predict a significant proportion of the variance on the Algebra ECA, will be explored. Finally, the results of the third regression model will be analyzed to determine if the eighth-grade ISTEP+ language arts test could be used to predict a significant proportion of the variance on the Algebra ECA. Each section will discuss the results for all four cohorts described in Chapter 3, including the 2010 advanced cohort, the 2010 average cohort, the 2011 advanced cohort, and the 2011 average cohort.

### Descriptive Statistics

The data were separated into four groups, and three analyses were conducted for each group. The first group, referred to as the 2010 advanced cohort ($N = 451$), consisted of the scores of students who took both the ISTEP+ and the Algebra ECA in the spring of 2010, their eighth-grade year. The 2010 average cohort consisted of the scores of students ($N = 264$) who took the ISTEP+ in the spring of 2010 and the Algebra ECA in the spring of 2011. The third group, referred to as the 2011 advanced cohort (*N* = 254), consisted of scores of students who took both the ISTEP+ and the Algebra ECA in the spring of 2011. The scores of students who took the ISTEP+ in the spring of 2011 and the Algebra ECA in the spring of 2012 formed the final group, referred to as the 2011 average cohort (*N* = 321). Table 1 shows the mean and standard deviation for each group on each test.

Table 1

*Means and Standard Deviations for Cohorts*

| | ISTEP+ Math Test | | | ISTEP+ ELA Test | | | Algebra ECA | | |
|---|---|---|---|---|---|---|---|---|---|
| Cohort | *M* | *SD* | *N* | *M* | *SD* | *N* | *M* | *SD* | *N* |
| 2010 | | | | | | | | | |
| Advanced | 590.26 | 53.46 | 452 | 556.23 | 57.17 | 451 | 565.45 | 67.40 | 452 |
| Average | 543.98 | 44.33 | 265 | 514.00 | 49.37 | 264 | 575.08 | 62.58 | 265 |
| 2011 | | | | | | | | | |
| Advanced | 637.26 | 49.18 | 255 | 591.56 | 60.05 | 254 | 657.41 | 53.75 | 255 |
| Average | 569.78 | 42.85 | 321 | 520.84 | 41.94 | 321 | 578.63 | 60.74 | 321 |

*Note.* ISTEP+, Indiana State Test of Education Progress Plus; *N*, number of students; ELA, English Language Arts; ECA, End-of-Course Assessment.

**Model 1: Eighth-Grade ISTEP+ Mathematics Score as Predictor of Algebra Success**

The first analysis was conducted to determine if the eighth-grade ISTEP+ mathematics score could be used to predict a significant proportion of the variance on the Algebra ECA. The purpose of the study is to determine what prior knowledge might be the best predictor of success in algebra as measured by the Algebra ECA. Prior to analyzing specific standards it was necessary to determine if the overall mathematics ISTEP+ test score would be a significant predictor. This analysis was conducted for each of the four groups previously specified, and the results are reported in Table 2. In each case, the null hypothesis was rejected.

Table 2

*ISTEP+ Mathematics Score as Predictor of Algebra End-of-Course Assessment*

| | 2010 | | 2011 | |
|---|---|---|---|---|
| Variable | Advanced | Average | Advanced | Average |
| ISTEP+ Math Score (*B*) | 1.05 | 0.80 | 0.77 | 0.69 |
| Number of students (*N*) | 453 | 261 | 253 | 315 |
| $R^2$ | 0.68 | 0.26 | 0.49 | 0.30 |
| Adjusted $R^2$ | 0.67 | 0.26 | 0.49 | 0.30 |
| *F* | 935.69* | 93.36* | 243.79* | 134.94* |

*Note*. *$p < 0.001$.

**Model 1 Analysis of the 2010 Advanced Cohort**

The data for the 2010 advanced cohort represented the scores of students who took both the ISTEP+ and the Algebra ECA in the spring of 2010, the end of their eighth-grade year. First, a scatterplot was created using the ISTEP+ math score as the predictor variable and the Algebra ECA score as the dependent variable. Although there was nothing to indicate non-linearity in this model, a few outliers were noted. When looking at the case-wise diagnostics, two cases in particular seemed extreme. In the first case, the ISTEP+ score was extremely low compared to the Algebra ECA score, indicating that the student might not have given his or her best effort on ISTEP+ test. In the second extreme case, the student obtained only the minimum score of 300 on the Algebra ECA, perhaps indicating that the student did not make a valid attempt on that test. Thus, both extreme scores were eliminated from the cohort.

After the removal of these two cases, the regression model predicting the Algebra ECA score from the eighth-grade ISTEP+ mathematics score was significant, $F(1, 451) = 935.69$, $p < 0.001$ (see Table 2). Specifically, 68% of the variance in Algebra ECA scores can be explained by the ISTEP+ math score $R^2 = 0.68$, Adj. $R^2 = 0.67$. According to the model, for every point increase in the ISTEP+ math score, the Algebra ECA score is expected to increase by 1.05 points, $b = 1.05$. Upon examination of the scatterplot of the Math ISTEP+ and the Algebra ECA, the assumption of linearity appeared to be met. The normal probability plot and histogram of the residuals indicate normality of residuals. Homoscedasticity is evidenced in the plot of residuals.

**Model 1 Analysis of the 2010 Average Cohort**

The data for the 2010 average cohort represented the scores of students who took the ISTEP+ math test in the spring of 2010 and the Algebra ECA in the spring of 2011, upon completion of algebra in ninth grade. After analysis of the initial scatterplot, three clear outliers were evident. In each case, the ISTEP+ math score was so low that it would seem reasonable to conclude that the students did not give their best efforts. Thus, these three cases were eliminated from the cohort.

After the removal of these cases, the regression model predicting the Algebra ECA score from the eighth-grade ISTEP+ mathematics score was significant, $F(1, 261) = 93.36$, $p < 0.001$. Specifically, only 26% of the variance in Algebra ECA scores can be explained by the ISTEP+ math score, and there was no change when adjusted for sample size and number of predictors, $R^2 = 0.26$, Adj. $R^2 = 0.26$. According to the model, for every point increase in the ISTEP+ math score, the Algebra ECA score is expected to increase by 0.80 points, $b = 0.80$ (see Table 2).

Upon examination of the scatterplot of the Math ISTEP+ and the Algebra ECA, the assumption of linearity appeared to be met. The normal probability plot and histogram of the residuals indicate normality of residuals. Homoscedasticity is evidenced in the plot of residuals.

**Model 1 Analysis of the 2011 Advanced Cohort**

The data for the 2011 advanced cohort represented the scores of students who took the ISTEP+ math test in the spring of 2011 and the Algebra ECA in the spring of 2011. The regression model predicting the Algebra ECA score from the eighth-grade ISTEP+ mathematics score was significant, $F(1, 253) = 243.79$, $p < 0.001$. Specifically, 49% of the variance in Algebra ECA scores can be explained by the ISTEP+ math score, and there was no change when adjusted for sample size and number of predictors, $R^2 = 0.49$, Adj. $R^2 = 0.49$. According to the model, for every point increase in the ISTEP+ math score, the Algebra ECA score is expected to increase by 0.77 points, $b = 0.77$ (see Table 2). Upon examination of the scatterplot of the Math ISTEP+ and the Algebra ECA, the assumption of linearity appeared to be met. The normal probability plot and histogram of the residuals indicate normality of residuals. Homoscedasticity is evidenced in the plot of residuals.

**Model 1 Analysis of the 2011 Average Cohort**

The data for the 2011 average cohort represented the scores of students who took the ISTEP+ math test in the spring of 2011 and the Algebra ECA in the spring of 2012, upon completion of algebra in ninth grade. After analysis of the initial scatterplot, four clear outliers were evident. In each case, the score on the Algebra ECA was so low that it seemed reasonable to conclude that the students did not give their best effort. Thus, these four cases were eliminated from the cohort.

After the removal of these cases, the regression model predicting the Algebra ECA score from the eighth-grade ISTEP+ mathematics score was significant, $F(1, 315) = 134.94$, $p < 0.05$. Specifically, 30% of the variance in Algebra ECA scores can be explained by the ISTEP+ math score, and there was no change when adjusted for sample size and number of predictors, $R^2 = 0.30$, Adj. $R^2 = 0.30$. According to the model, for every point increase in the ISTEP+ math score, the Algebra ECA score is expected to increase by 0.69 points, $b = 0.69$ (see Table 2). Upon examination of the scatterplot of the Math ISTEP+ and the Algebra ECA, the assumption of linearity appeared to be met. The normal probability plot and histogram of the residuals indicate normality of residuals. However, there is some evidence of heteroscedasticity in the plot of the residuals, which may limit the generalizability of this model.

The first analysis indicated that the regression model generated for each group was significant, although the amount of variance explained by each model varied widely from group to group. However, it is safe to conclude that the eighth-grade ISTEP+ math score can be used to predict a significant proportion of the variance on the Algebra ECA and the null hypothesis is rejected. Further, as the assumptions have been met, these results are generalizable to the population at large. Having established that the ISTEP+ math test can be used to predict success on the Algebra ECA, the next step was to determine if specific standards could be linked to success in Algebra. The following section reports the analysis of this second model.

**Model 2: Eighth-Grade ISTEP+ Mathematics Individual Reporting Categories as Predictors of Algebra Success**

The second research question in this study asked if the seven individual reporting subcategories on the eighth-grade ISTEP+ math test could be used to predict a significant proportion of the variance on the Algebra ECA. These reporting subcategories are number sense, computation, algebra and functions, geometry, measurement, data analysis and probability, and problem solving. This question is important because it sought to determine which of these categories might contribute more to success on the Algebra ECA. If educators understand exactly what kind of prior knowledge helps students learn algebra, then teaching and remediation efforts could be focused in this area.

To answer this question, a multiple regression analysis was conducted for each of the four groups, using each of the seven reporting subcategories as predictor variables. However, when the data were initially analyzed for each cohort, it was noted that there was a high intercorrelation between many of the individual predictor variables, with the greatest values coming from the subcategory of problem solving. According to IDOE (2012c), problem solving is always assessed on the test along with another subcategory. All of the problem-solving questions are in the open-response portion of the test given in the early spring, and they are all reported as *problem solving and algebra and functions* or *problem solving and geometry*. Each question with problem solving was really being counted in the score twice, once in problem solving and once in the associated subcategory being tested. Thus, to avoid redundancy and solve the issue of multicollinearity, it was prudent to remove the subcategory of problem solving from the multiple regression analysis in each cohort.

Once problem solving was removed from the analysis, the analysis indicated that the multiple regression equation generated for each group was statistically significant. Thus, the null hypothesis was rejected. Table 3 summarizes the results for each group and the following narrative describes the results, after removing the reporting subcategory of problem solving.

Table 3

*ISTEP+ Mathematics Individual Reporting Subcategory Scores as Predictors of Algebra End-of-Course Assessment Scores*

| Variable | 2010 Advanced | 2010 Average | 2011 Advanced | 2011 Average |
|---|---|---|---|---|
| Number sense (*B*) | 0.29 | 0.43 | -0.06 | 0.65 |
| Computation (*B*) | 0.37 | 0.43 | 0.09 | 0.32 |
| Algebra and functions (*B*) | 1.13** | 1.60* | 1.96** | 0.93* |
| Geometry (*B*) | 0.20 | 0.54 | 0.49 | -0.07 |
| Measurement (*B*) | 0.52* | 0.11 | 0.44 | 0.51 |
| Data analysis (*B*) | 0.59* | 0.41 | -0.56 | -0.17 |
| *N* | 446 | 255 | 248 | 310 |
| $R^2$ | 0.69 | 0.26 | 0.50 | 0.32 |
| Adjusted $R^2$ | 0.69 | 0.24 | 0.49 | 0.30 |
| *F* | 166.51** | 14.73** | 41.22** | 23.71** |

*Note.* $*p < 0.05$, $**p < 0.001$.

**Model 2 Analysis of the 2010 Advanced Cohort**

The data for this analysis consisted of the scores from the six individual reporting subcategories (number sense, computation, algebra and functions, geometry, measurement, and data analysis and probability) as the predictor variables and the Algebra ECA score as the dependent variable. Again, these scores came from students who took both the ISTEP+ math test and the Algebra ECA in the spring of 2010, the end of their eighth-grade year. This analysis included the same group as the first model, sans outliers. First, a matrix scatterplot was created which revealed nothing to indicate non-linearity. When predicting the Algebra ECA from the six predictors, it was noted that the model was significant, $F(6, 446) = 166.51$, $p < 0.001$, explaining 69% of the variance in test scores, $R^2 = 0.69$, Adj. $R^2 = 0.69$ (see Table 3). However, not all of the predictor variables were significant.

In this multiple regression analysis, only the categories of algebra and functions, measurement, and data analysis and probability were significant. Specifically, for every additional point in the algebra and functions subcategory on the ISTEP+ exam, a student's Algebra ECA score is expected to increase by 1.13 points, holding all other variables constant, $t(446) = 3.99$, $p < 0.001$. Likewise, for every additional point in the measurement subcategory on the ISTEP+ exam a student's Algebra ECA score is expected to increase by 0.52, holding all other variables constant, $t(446) = 2.71$, $p < 0.05$. For every additional point a student earns on the data analysis and probability subcategory, a student's Algebra ECA score is expected to increase by 0.59 points, holding all other variables constant, $t(446) = 2.49$, $p < 0.05$.

With this model there was still evidence of multicollinearity. However, this is not surprising given the nature of the variables and the interconnectedness of the math content. Nevertheless, the tolerance values were all between 0.1 and 0.2, indicating a potential problem but meeting minimum requirements (Field, 2009). Likewise, although the average of the variance inflation factor (VIF) values is greater than 1, each individual value is less than 10, again meeting minimum criteria (Field, 2009.) Additionally, all other assumptions were checked. The normal probability plot and histogram of the residuals indicate normality of residuals. Homoscedasticity is evidenced in the plot of residuals.

**Model 2 Analysis of the 2010 Average Cohort**

The data for this analysis came from the scores of students who took the ISTEP+ math test in the spring of 2010 and the Algebra ECA in 2011, upon completion of algebra in ninth grade. First a matrix scatterplot was created which revealed nothing to indicate non-linearity. It was noted that the model was significant, $F(6, 255) = 14.73$, $p < 0.001$, explaining only 26% of the variance in test scores, $R^2 = 0.26$, Adj. $R^2 = 0.24$.

However, only one of the predictor variables, algebra and functions, was significant (see Table 3). Specifically, for every additional point in the algebra and functions subcategory on the ISTEP+ exam, a student's Algebra ECA score is expected to increase by 1.60 points, holding all other variables constant, $t(255) = 2.92$, $p < 0.05$. Again, with this model, as with the previous one, there was still evidence of multicollinearity. However, it was determined that the minimum criterion for tolerance and VIF levels was met (Field, 2009). Additionally, all other assumptions were checked. The normal probability plot and histogram of the residuals indicate normality of residuals, and homoscedasticity is evidenced in the plot of residuals.

**Model 2 Analysis of the 2011 Advanced Cohort**

The data for this analysis came from the scores of students who took both the ISTEP+ math test and the Algebra ECA in the spring of 2011. A matrix scatterplot was created which revealed nothing to indicate non-linearity. The regression demonstrated a significant model, $F(6, 248) = 41.22$, $p < 0.001$, explaining 50% of the variance in test scores, $R^2 = 0.50$, Adj. $R^2 = 0.49$. As with the previous group, only one of the predictor variables, algebra and functions, was significant (see Table 3). Specifically, for every additional point in the algebra and functions subcategory on the ISTEP+ exam, a student's Algebra ECA score is expected to increase by 1.96 points, holding all other variables constant, $t(248) = 4.59$, $p < 0.001$.

Again, with this model, as with the previous one, there was still evidence of multicollinearity. However, it was determined that the minimum criterion for tolerance and VIF levels was met (Field, 2009). Additionally, all other assumptions were checked. The normal probability plot and histogram of the residuals indicate normality of residuals, and homoscedasticity is evidenced in the plot of residuals.

**Model 2 Analysis of the 2011 Average Cohort**

The data for this analysis came from the scores of students who took the ISTEP+ math test in the spring of 2011 and the Algebra ECA in the spring of 2012. A matrix scatterplot was created which revealed nothing to indicate non-linearity. The regression model generated was significant, $F(6, 310) = 23.71$, $p < 0.001$, explaining 32% of the variance in test scores, $R^2 = 0.32$, Adj. $R^2 = 0.30$, although only one of the predictor variables, algebra and functions, was significant (see Table 3). Specifically, for every additional point in the algebra and functions subcategory on the ISTEP+ exam, a student's Algebra ECA score is expected to increase by 0.93 points, holding all other variables constant, $t(310) = 2.52$, $p < 0.05$.

Again, with this model, as with the previous one, there was still evidence of multicollinearity. However, it was determined that the minimum criterion for tolerance and VIF levels was met. Additionally, all other assumptions were checked. The normal probability plot and histogram of the residuals indicate normality of residuals, and homoscedasticity is evidenced in the plot of residuals.

**Model 3: Eighth-Grade ISTEP+ Language Arts Score as Predictor of Algebra Success**

The final analysis was conducted in order to determine if the ISTEP+ language arts score might predict success on the Algebra ECA. This analysis was introduced based on the work of Liang (2009), who found in a similar study of standardized test scores in California that the language arts score was one of the leading predictors of algebra success. Again, if it were determined that the ability to do well in language arts was connected to the ability to succeed in algebra, it might help educators understand how better to appropriate resources for instruction and remediation. The regression equation generated for each group proved significant, and the null hypothesis was rejected. Table 4 summarizes the results for each group and the following sections report the regression models and analysis.

Table 4

*ISTEP+ Language Arts Score as Predictor of Algebra End-of-Course Assessment Score*

| | 2010 | | 2011 | |
|---|---|---|---|---|
| Variable | Advanced | Average | Advanced | Average |
| ISTEP+ ELA Score (*B*) | 0.66 | 0.26 | 0.36 | 0.35 |
| *N* | 449 | 260 | 252 | 315 |
| $R^2$ | 0.32 | 0.04 | 0.17 | 0.07 |
| Adjusted $R^2$ | 0.32 | 0.04 | 0.16 | 0.07 |
| *F* | 207.72* | 11.96* | 49.81* | 24.60* |

*Note.* *$p < 0.001$.

**Model 3 Analysis of the 2010 Advanced Cohort**

The data for this analysis came from the scores of students who took both the ISTEP+ language arts test and the Algebra ECA in the spring of 2010. A scatterplot was created to test the linearity of the model, and there was nothing to indicate non-linearity. The regression model generated was significant, $F(1, 449) = 207.72$, $p < 0.05$, explaining 32% of the variance in test scores, $R^2 = 0.32$, Adj. $R^2 = 0.32$. According to the model, for every point increase in the ISTEP+

language arts score, the Algebra ECA score is expected to increase by 0.66 point, $b = 0.66$. Upon examination of the scatterplot of the ISTEP+ language arts test and the Algebra ECA, the assumption of linearity appeared to be met. The normal probability plot and histogram of the residuals indicate normality of residuals. Homoscedasticity is evidenced in the plot of residuals. A few outliers were noted, but none seemed extreme enough to warrant removal from the cohort.

**Model 3 Analysis of the 2010 Average Cohort**

The data for this analysis came from the scores of students who took the ISTEP+ language arts test in the spring of 2010 and the Algebra ECA in the spring of 2011. A scatterplot was created to test the linearity of the model, which called into question the assumption of linearity. The regression demonstrated a significant model, $F(1, 262) = 11.96$, $p < 0.05$, although it only explained a mere 4% of the variance in test scores, $R^2 = 0.04$, Adj. $R^2 = 0.04$. According to the model, for every point increase in the ISTEP+ language arts score, the Algebra ECA score is expected to increase by 0.26 point, $b = 0.26$. The normal probability plot and histogram of the residuals indicate normality of residuals. Homoscedasticity is evidenced in the plot of residuals. Although significant, this model does not explain much of the variance in scores, and thus is not very relevant.

**Model 3 Analysis of the 2011 Advanced Cohort**

The data for this analysis came from the scores of students who took both the ISTEP+ language arts test and the Algebra ECA in the spring of 2011. A scatterplot was created to test the linearity of the model, and there was nothing to indicate non-linearity. Again, the regression model generated was significant, $F(1, 252) = 49.81$, $p < 0.05$, explaining 17% of the variance in test scores, $R^2 = 0.17$, Adj. $R^2 = 0.16$. According to the model, for every point increase in the ISTEP+ language arts score, the Algebra ECA score is expected to increase by 0.36 point, $b =$

0.36. Upon examination of the scatterplot of the ISTEP+ language arts test and the Algebra ECA, the assumption of linearity appeared to be met. The normal probability plot and histogram of the residuals indicate normality of residuals. Homoscedasticity is evidenced in the plot of residuals. A few outliers were noted, but none seemed extreme enough to warrant removal from the cohort.

**Model 3 Analysis of the 2011 Average Cohort**

The data for this analysis came from the scores of students who took the ISTEP+ language arts test in the spring of 2011and the Algebra ECA in the spring of 2012. A scatterplot was created to test the linearity of the model, and there was nothing to indicate non-linearity. The regression model generated was significant, $F(1, 315) = 24.60$, $p < 0.05$, explaining only 7% of the variance in test scores, $R^2 = 0.07$, Adj. $R^2 = 0.07$. According to the model, for every point increase in the ISTEP+ language arts score, the Algebra ECA score is expected to increase by 0.35 point, $b = 0.35$. Upon examination of the scatterplot of the ISTEP+ language arts test and the Algebra ECA, the assumption of linearity appeared to be met. The normal probability plot and histogram of the residuals indicate normality of residuals. Homoscedasticity is evidenced in the plot of residuals. A few outliers were noted, but none seemed extreme enough to warrant removal from the cohort.

## Summary

Three regression models were completed for each of the four groups in an attempt to answer the research questions. Each of these models was significant, although the amount of variance explained varied greatly. Additionally, all assumptions were met for each model, at least at a minimum level. When analyzing the multiple regression models to determine if the reporting subcategories for the ISTEP+ math test could be used to predict a significant proportion of the variance on the Algebra ECA, it was noted that there was evidence of multicollinearity. In an effort to combat this, the subcategory of problem solving was removed. The removal of this subcategory did result in the model meeting the minimum requirements, although some mulitcollinearity was still evident. However, this evidence was not surprising given the nature of the data and the relationship between the different mathematics skills being tested on the ISTEP+ math test. Chapter 5 will discuss the results of this analysis, draw conclusions, and make recommendations for future study.

# CHAPTER 5

# DISCUSSION

It has been established that algebra is a basic requirement for high school graduation. In the state of Indiana, students need to earn two credits of algebra and must pass the Algebra ECA to meet graduation requirements. Additionally, algebra is the introductory course for all higher-level mathematics, serving as a gateway for further education and careers. Yet, too many students are not experiencing success early enough in this critical course, thereby limiting their chances to pursue higher education and more lucrative job opportunities. Educators need to find a way to help more students achieve success in this critical course.

Using a quantitative approach, this study sought to determine the specific prerequisite knowledge that students need in order to pass the Algebra ECA. Using both linear and multiple regression analyses, the eighth-grade ISTEP+ test and the specific reporting subcategories in the math portion of the test were utilized to predict a significant proportion of the variance on the Algebra ECA. In particular, if certain reporting subcategories could predict more of the variance in scores, educators would know how to best remediate and prepare students for success in algebra.

## Discussion of the Findings

The first research question asked if the eighth-grade ISTEP+ mathematics test composite scores could be used to predict the Algebra ECA score. Two cohorts of students were used in the analysis, with each cohort divided into advanced and average cohorts. It was determined for all four cohorts that the eighth-grade ISTEP+ mathematics test composite score was a significant predictor of success on the Algebra ECA, leading to a rejection of the null hypothesis. However, the variance explained varied greatly from group to group.

For the 2010 advanced cohort (*N* = 451), including those students who took both assessments in the spring of their eighth-grade year, the linear regression model explained 68% of the variance. However, this sharply contrasts with the 2010 average cohort (*N* = 261) where the model explained only 26% of the variance. Similarly, though not as extreme, the linear regression model for the 2011 advanced cohort (*N* = 253) explained 49% of the variance as compared to only 30% of the variance for the 2011 average cohort (*N* = 315).

Perhaps the larger amount of variance explained by the models for both advanced cohorts is an indication that they were more homogeneous in their ability. The students in these two cohorts were selected based on previous performance to take algebra in eighth grade. The Average cohorts represent students who did not take algebra until ninth grade. It could be assumed that there was a wider range of abilities represented in this group, perhaps contributing to the fact that the models explained less of the variance. However, different tests were also used in each model, which could also contribute to different amounts of variance explained.

Additionally, it is important to note a rather significant change in the size of the sample population. The 2010 advanced cohort had 451 participants whereas the 2011 advanced cohort had only 253 participants. This difference reflects a change in district policy regarding which students should take algebra in eighth grade. In 2010, more students were encouraged to take algebra, but according to the results reported by IDOE, only 50% of those students passed the Algebra ECA. Thus, the district decided to limit the number of students who took algebra in eighth grade. As a result, the number of algebra students the following year in that district was much lower, but the percentage of those students passing the Algebra ECA was 94%. Additionally, the 2010 average cohort consisted of 261 participants, and they passed the Algebra ECA at a rate of 59%. However, after keeping more students in eighth-grade math in 2011, the 2011 average cohort passed the Algebra ECA at a rate of 68%. This more conservative approach to placing students in algebra seemed to benefit the students, with more students experiencing success on the Algebra ECA.

For the 2010 advanced cohort, the estimated regression model was Y' = -51.98 + 1.05(ISTEP+ score). According to IDOE, a score of 564 on the Algebra ECA is considered passing and a score of 537 on the eighth-grade ISTEP+ mathematics test is considered passing. The model generated for the 2010 advanced cohort predicts that a student would have needed to score a 586 on the eighth-grade ISTEP+ in order to earn a passing score on the Algebra ECA, a score approximately 50 points higher than the minimum passing score. This model would indicate that merely passing the ISTEP+ mathematics test is not enough to ensure success on the Algebra ECA.

However, the results for the 2010 average cohort were quite different. The regression equation for this model was Y' = 141.71 + 0.80(ISTEP+ score). According to this equation, a score of only 527 on the ISTEP+ mathematics test would predict success on the Algebra ECA. This score represents a score that is approximately 10 points below passing, thus indicating quite different results.

For the 2011 advanced cohort, the results were similar. The regression equation for this model was Y' = 169.5 + 0.77(ISTEP+ score). According to this equation, a score of 515 on the ISTEP+ mathematics test would predict success on the Algebra ECA. Again, this score of 515 is below passing on the ISTEP+, though only by about 22 points. The regression equation produced by the model for the 2011 average cohort was Y' = 187 + 0.69(ISTEP+ score). According to this model, a score of 545 on the ISTEP+ mathematics test would predict success on the Algebra ECA. This score is less than 10 points above the cut score for the ISTEP+ mathematics test. Thus, it is evident that the results for all four models are very mixed. However, it can be concluded that students who pass the eighth-grade ISTEP+ mathematics test are more likely to pass the Algebra ECA, and the corresponding null hypothesis was rejected.

The second research question asked if the scores on the seven individual reporting subcategories tested on the ISTEP+ could be used to predict a significant proportion of the variance on the Algebra ECA. These categories consist of number sense, computation, algebra and functions, geometry, measurement, data analysis and probability, and problem solving. A multiple regression analysis was conducted for each of the four cohorts. However, upon the initial analysis there was evidence of multicollinearity. In order to alleviate this problem, the subcategory of problem solving was removed, as it was assessed in conjunction with two of the other subcategories. After removing problem solving, the analyses met the minimum requirements necessary to proceed.

In each case, the models were statistically significant, allowing for the rejection of the null hypothesis, although again the results varied widely from model to model. According to the multiple regression analysis of the 2010 Advanced cohort, only the subcategories of algebra and function, measurement, and data analysis were significant. For the 2010 average cohort, only the subcategory of algebra and functions was statistically significant, and the same was true for both the 2011 advanced and average cohorts. These results seem to indicate that those students who were able to master the algebra and functions assessed in eighth grade were subsequently able to succeed in algebra in high school.

According to the *ISTEP+: Grade 8 Mathematics Blueprint* (IDOE, 2012c), the subcategory of algebra and functions is one of the largest portions of the test, consisting of 15% to 25% of the test. This large proportion of the test could be yet another reason why this portion was consistently determined to be significant in the model. Additionally, this portion of the test has many similarities to questions that could also be asked on the Algebra ECA. According to the *Indiana Mathematics Grade 8 Curriculum Framework* (IDOE, 2002), which was in effect for the testing during this period of study, there are 10 different standards that fall under the subcategory of algebra and functions, including writing and solving linear equations and inequalities in one variable, solving systems of linear equations using substitution and graphing, and graphing linear functions and identifying slope. All of these standards are also included in the algebra standards, although the algebra standards take each a step further. For example, while the eighth-grade standards only indicate that students should be able to solve systems of equations by substitution, the algebra standards add the ability to solve systems by addition, subtraction, multiplication, and division. The algebra standards also specifically mention the ability to solve word problems.

A review of the literature had indicated a link between arithmetic and the ability to learn algebra. Based on this, one might have expected the categories of number sense and computation to be significant. However, they were not found to be significant predictors in any of the four models. A review of the eighth-grade mathematics standards in the subcategory of number sense reveals why this might not have been statistically significant. According to IDOE (2012c), number sense is only 8% to 18% of the assessment. Additionally, all of these standards generally relate to understanding rational and irrational numbers, along with exponents, powers, and roots. A review of the literature did not reveal any connection between these particular skills and success in algebra.

Although a review of the literature seemed to indicate a link between arithmetic and algebra, none of the models generated found computation to be a statistically significant predictor of success in algebra. However, the subcategory of computation is one of the smallest portions of the test, consisting of 5% to 15% of the questions (IDOE, 2012c). A review of the eighth-grade standards indicates that these standards include operations with rational numbers and computing simple interest. Some have tried to link the ability to work with fractions and success in algebra, but this test would not seem to assess fractions enough to determine if this would be relevant.

The third research question sought to determine if the score on the eighth-grade language arts ISTEP+ exam could be used to predict a significant proportion of the variance on the Algebra ECA. A review of the literature seemed to indicate that this could be a significant predictor variable. The results were statistically significant, leading to the rejection of the corresponding null hypothesis. However, in each of the four models produced by the linear regression model, only a small percent of the variance was explained. For the 2010 advanced cohort, the eighth-grade language arts score explained 32% of the variance. However, for the 2010 average cohort, the model explained a mere 4% of the variance in the Algebra ECA test scores. For the 2011 advanced cohort, the eighth-grade language arts score explained 17% of the variance in Algebra ECA scores and for the average cohorts the model explained only 7% of the variance.

## Educational Implications

As educators plan their instructional calendars, it would be beneficial for them to know that the algebra and functions portion of the eighth-grade ISTEP+ mathematics exam seems to be the best predictor of success on the Algebra ECA. The largest portion of time should be given to algebra and functions in eighth grade. Additionally, it is important to note that remediation should not start in ninth grade. By that time, it is too late. Remediation needs to occur in middle school, when students first begin having trouble with algebra.

## Limitations

Limitations were found in this study. First, all of the data came from one school district. It is a large, diverse district, but all students attended school at one of three middle schools and the same high school. The district could have particular curriculum calendars and instructional practices that contributed to the results on the test. A larger, more diverse population sample may have yielded better, more generalizable results.

Additionally, data from two years of testing were considered in this study, whereas it might have been more beneficial to include a third year. A change in district policy between the years of study regarding which students were allowed to take algebra in eighth grade made it difficult to compare the two cohorts. In the 2009–2010 school year, the district had a more liberal policy regarding eighth-grade algebra, allowing a larger number of students to take the class who may have not been ready. However, the following year, this policy was changed, limiting the number of eighth-grade algebra students to a more select group.

## Future Research

Because many algebra topics tested on the Algebra ECA are introduced in the eighth-grade standards, it would be beneficial to see if ISTEP+ tests from earlier grades could be used to predict success on the Algebra ECA. Using earlier ISTEP+ testing might help better determine which skills really do contribute to one's ability to learn algebra. For example, a students' ability to fluently work with fractions is frequently considered critical to algebra success. Fractions are typically taught in fourth and fifth grades, so using these ISTEP+ tests to predict success on the Algebra ECA might be more beneficial. Much of the research indicated a link between algebra and arithmetic, but arithmetic was not really assessed on the eighth-grade ISTEP+. Thus, this would be another reason to consider looking at the content strands of other grade levels to predict algebra success.

Additionally, when the eighth-grade ISTEP+ language arts score was used to predict the Algebra ECA score, not much of the variance was explained. However, it might be more beneficial to consider a reading score and the Algebra ECA, as opposed to the language arts score. The eghth-grade language arts test assessed reading comprehension and writing skills. However, a focus on reading fluency might be a better predictor of success in algebra. Perhaps using scores from the Scholastic Reading Inventory or some other assessment used to measure reading fluency would yield stronger results. This is another area where future study might be warranted.

## Summary

In conclusion, research was conducted to determine if the eghth-grade ISTEP+ scores on the both mathematics and language arts portions of the test could be used to predict success on the Algebra ECA. More specifically, a multiple regression analysis was conducted to determine if one or more of the seven individual reporting subcategories might have stronger predictive qualities than the other. The analysis determined that the only reporting subcategory that was significant among all subgroups was algebra and functions. These results would indicate that it is critical for students to have early exposure to algebra, so they are prepared for algebra in high school. Further study should be conducted, perhaps using ISTEP+ tests from earlier grades to determine if a better predictor variable could be established. Additionally, further analysis should be conducted regarding reading fluency as opposed to the overall language arts score.

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